173. *Weight Functions of the Class* (A_∞) *and Quasi-conformal Mappings*

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(Comm. by Kôsaku YOSIDA, M. J. A., Nov. 12, 1975)

§ 1. **Introduction.** In the following we use G as an open subset of R^n, Q (or P) as a cube with sides parallel to coordinates axis, E as a measurable set and $\chi(E)$ as the characteristic function of E. When f is a measurable function defined on R^n, $\sup\left\{\left(|Q|^{-1}\int_Q |f(y)|^p\,dy\right)^{1/p} \,|\, Q \ni x\right\}$ will be denoted by $M_p(f)(x)$. If $\varphi: G_1 \to G_2$ is totally differentiable at x, the Jacobian matrix of φ at x will be denoted by $\Phi(x)$ and $|\det \Phi(x)|$ by $J_\varphi(x)$. For ACL (absolutely continuous on lines) and BMO (bounded mean oscillation) see Reimann [4].

In Reimann [4] he proved the following theorem.

Theorem A. *Let φ be a homeomorphism of R^n onto itself, ACL and totally differentiable a.e. and assume that $|\varphi(\cdot)|$ and $|\varphi^{-1}(\cdot)|$ are absolutely continuous set functions in R^n. Then φ is quasiconformal iff there exists $C>0$ such that $\|f\circ\varphi^{-1}\|_* \leqslant C\|f\|_*$ for any BMO function f, where $\|\cdot\|_*$ means the BMO norm.*

Using his idea, some other characterizations of quasiconformal mappings are possible. Theorem 1 and Corollary 1 are characterizations by Hardy-Littlewoods' maximal functions and Theorem 2 is a characterization by some kind of measures.

§ 2. **The Hardy-Littlewoods' maximal functions and quasiconformal mappings**

Theorem 1. *Let φ be a homeomorphism of G_1 onto G_2, ACL and totally differentiable a.e. Then the followings are equivalent.*

(I) *φ is a quasiconformal mapping.*

(II) *There exist $C>0$ and $\infty>p>1$ satisfying the following conditions:*

For $\forall x\in G_1$ there exists $r(x)>0$ such that

$$\sup\left\{|Q|^{-1}\int_Q f(y)dy \,\middle|\, \operatorname{diam} Q<r(x),\ Q\ni x\right\} \leqslant C\sup\left\{\left(|Q|^{-1}\int_Q (f\circ\varphi^{-1}(y))^p dy\right)^{1/p} \,\middle|\, Q\ni\varphi(x),\ Q\subset G_2\right\}, \tag{1}$$

$$\sup\left\{|Q|^{-1}\int_Q f\circ\varphi^{-1}(y)dy \,\middle|\, \operatorname{diam} Q<r(x),\ Q\ni\varphi(x)\right\} \leqslant C\sup\left\{\left(|Q|^{-1}\int_Q f(y)^p dy\right)^{1/p} \,\middle|\, Q\ni x,\ Q\subset G_1\right\} \tag{2}$$