61. On Higher Coassociativity

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(Comm. by Kenjiro SHODA, M. J. A., May 12, 1976)

In this note, we generalize the coassociativity of co-H-spaces and primitive maps among them, and give a relation between A'_4-spaces and their coretractions. We work in the category of based spaces having the homotopy types of CW-complexes and based maps. Details will appear in [2].

Let K_n $(n\geqq 2)$ be Stasheff's convex polyhedron [3], which admits face maps $\partial_k(r,s)\colon K_r\times K_s\to K_n$, $r+s=n+1$, $1\leqq k\leqq r$, and degeneracy maps $s_j\colon K_n\to K_{n-1}$, $1\leqq j\leqq n$, satisfying suitable FD-commutativities. For a given based space X, $W_n(X)$ denotes the wedge product of n-copies of X, (i,x) denotes the element whose i-th factor is x.

Definition 1. A space X is an *A'_n-space*, if there exists an *A'_n-structure* $\{M'_{X,i}\colon X\times K_i\to W_i(X)\}_{2\leqq i\leqq n}$ satisfying the following conditions:

(1.1) $\mu'_X=M'_{X,2}$ is a comultiplication with the counit $*\colon X\to *$, where $*$ is the base point of X;

(1.2) for any $(\rho,\sigma)\in K_r\times K_s$, $r+s=i+1$, it holds

$$M'_i(\ ;\partial_k(r,s)(\rho,\sigma))=M'_s(\ ;\sigma)(k)\cdot M'_r(\ ;\rho),$$

where $M'_s(\ ;\sigma)(k)$ implies $M'_s(\ ;\sigma)$ is applied on the k-th factor and 1 is applied on other factors;

(1.3) for $i\geqq 3$, there exist homotopies

$$D'_{X,i,j}\colon M'_{i-1}(\ ;s_j(\tau))\simeq p_j M'_i(\ ;\tau)$$

where $p_j=\nabla(j)\cdot *(j)$ and $\nabla\colon X\vee X\to X$ is the folding map.

Definition 2. An A'_n-space X $(n\geqq 3)$ is an *A'_n-cogroup*, if there exists a coinversion $\nu'_X\colon X\to X$ such that it holds $\nabla\cdot(1\vee\nu'_X)\cdot\mu'_X\simeq *\simeq\nabla\cdot(\nu'_X\vee 1)\cdot\mu'_X$.

Definition 3. A map $f\colon X\to Y$ of A'_n-spaces is an *A'_n-map* if there exist homotopies $H'_i\colon X\times K_i\times I\to W_i(Y)$, $2\leqq i\leqq n$, such that

(3.1) $H'_i((x;\tau),0)=W_i(f)\cdot M'_{X,i}(x;\tau)$

and

$$H'_i((x;\tau);1)=M'_{Y,i}(f(x);\tau);$$

(3.2) for any $\partial_k(r,s)$, $r+s=i+1$, $1\leqq k\leqq r$,

there exists a homeomorphism $\tilde{\partial}_k(r,s)$ of $K_r\times K_s\times I$ into $\partial K_i\times I$ which preserves level and satisfies