58. A Family of Pseudo-Differential Operators and a Stability Theorem for the Friedrichs Scheme

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§0. Introduction. In this note we shall study an algebra of a family of pseudo-differential operators and try to apply this theory to the stability theory of the Friedrichs scheme. The class $\{S^m_{\lambda_h}\}$ of pseudo-differential operators is defined by a family of basic weight functions $\lambda_h(\xi)$ $(0<h<1)$ as in [4], [5] and [2].

For the application to the stability theory we have to define two subclasses $\{\mathring{S}^m_{\lambda_h}\}$ and $\{\tilde{S}^m_{\lambda_h}\}$ of $\{S^m_{\lambda_h}\}$ as the sets of all the symbols $p_h(x,\xi)$ such that $h^{-1}p_h\in\{S^{m+1}_{\lambda_h}\}$ and $h^{-1}\partial^\alpha_\xi p_h\in\{S^{m+1-|\alpha|}_{\lambda_h}\}$ for any $\alpha\neq 0$, respectively. We have also to derive 'the principle of cutting off' a symbol $p_h(x,\xi)$ of class $\{S^m_{\lambda_h}\}$ by $\chi(\lambda_h(\xi))$ (or $\varphi(\zeta_h(\xi))$) (see Theorem 1.9). Then, we can treat difference schemes as a family of pseudo-differential operators, and prove a stability theorem of the Friedrichs schemes for a diagonalizable hyperbolic system. We note that this theorem is regarded as the general form of the Yamaguti-Nogi-Vaillancourt stability theorem in [7], [8] and [9], and note that the theorem holds without the restriction on the behavior of symbols $p_h(x,\xi)$ at $x=\infty$.

§1. A family of pseudo-differential operators.

Definition 1.1. A family $\{\lambda_h(\xi)\}_{0<h<1}$ of real valued C^∞-functions in R^n is called a basic weight function, when there exist positive constants A_0, A_α (independent of $0<h<1$) such that

(1.1) $1\leqq\lambda_h(\xi)\leqq A_0\langle\xi\rangle,\ |\lambda^{(\alpha)}_h(\xi)|\leqq A_\alpha\lambda_h(\xi)^{1-|\alpha|}$ for any α,

where $\langle\xi\rangle=\{1+|\xi|^2\}^{1/2}$, $\lambda^{(\alpha)}_h=\partial^\alpha_\xi\lambda_h$ for $\alpha=(\alpha_1,\cdots,\alpha_n)$.

Example. An important example of this note is defined by

(1.2) $\lambda_h(\xi)=\langle\zeta_h(\xi)\rangle$, $\zeta_h(\xi)=(h^{-1}\sin h\xi_1,\cdots,h^{-1}\sin h\xi_n)$ (see [4], [5]).

Definition 1.2. i) A family $\{p_h\}$ of C^∞-symbols $p_h(x,\xi)$ in $R^n_x\times R^n_\xi$ $(0<h<1)$ is called of class $\{S^m_{\lambda_h}\}$ $(-\infty<m<\infty)$, when there exist constants $C_{\alpha,\beta}$ (independent of $0<h<1$) such that

(1.3) $|p^{(\alpha)}_{h(\beta)}(x,\xi)|\leqq C_{\alpha,\beta}\lambda_h(\xi)^{m-|\alpha|}$ *for any* α,β,

where $p^{(\alpha)}_{h(\beta)}=\partial^\alpha_\xi D^\beta_x p_h$ $(D_x=-i\partial_x)$. We set $\{S^{-\infty}_{\lambda_h}\}=\bigcap_m\{S^m_{\lambda_h}\}$ and $\{S^\infty_{\lambda_h}\}=\bigcup_m\{S^m_{\lambda_h}\}$.

ii) A family $\{P_h\}$ of linear operators $P_h:\mathcal{S}\to\mathcal{S}$ is called a pseudo-differential operator of class $\{S^m_{\lambda_h}\}$ with symbol $p_h(x,\xi)$, when there

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