57. A Sharp Form of the Existence Theorem for Hyperbolic Mixed Problems of Second Order

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§ 1. **Introduction.** In this paper we consider the following initial boundary value problem

$$\{P, B\}\begin{cases} Pu=f(x,t), & \text{for } x\in\Omega,\ t>0, \\ Bu\big|_{\partial\Omega}=g(s,t) & \text{for } s\ni\partial\Omega,\ t>0, \\ D_t^j u\big|_{t=0}=u_j(x),\ (j=0,1), & \text{for } x\in\Omega, \end{cases}$$

in the cylindrical domain $\Omega\times(0,\infty)$, where Ω is the exterior or the interior of a smooth and compact hypersurface $\partial\Omega$ in R^{n+1}. P is a regularly hyperbolic operator with respect to t, and $\partial\Omega$ is non-characteristic to P. Moreover we assume that the only one of $\tau_1(\nu)$ and $\tau_2(\nu)$ is negative for all $(s,t)\in\partial\Omega\times(0,\infty)$, where $\tau_j(\xi)$ are the roots of $P(s,t;\xi,\tau)=0$ and ν is the inner unit normal at (s,t). This condition means that the number of boundary conditions is one. B is a first order operator:

$$B=B(s,t;D_x,D_t)=\sum_{j=1}^{n+1} b_j(s,t)D_{x_j}-c(s,t)D_t, \qquad D_t=\frac{1}{i}\frac{\partial}{\partial t} \qquad \text{etc.,}$$

where $\sum_{j=1}^{n+1} b_j(s,t)\nu_j=B(s,t,\nu,0)=1$. We assume that all the coefficients are smooth and bounded, and that they remain constant outside some compact sets.

We are concerned with the following question: Under what condition the solution $u(t)$ of $\{P,B\}$ has the continuity for the initial data in the same Sobolev space? The answer is just the condition (H) below, which was derived in [2].[1] We state it as

Theorem 1. *The necessary and sufficient condition that the energy inequality*

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1) (H) was introduced as a characterization of problems which satisfy

$$\gamma\,|u|_{1,\gamma}^2\leq\frac{c}{\gamma}|Pu|_{0,\gamma}^2,$$

holds for any smooth function with compact support satisfying the homogeneous boundary condition, in the case of constant coefficients. See also [1] and [3]. In [2] we proved the existence theorem with the initial data in a weaker sense. It is difficult to prove the estimate (1.1) as the direct extension of the arguments in [2]. For this purpose we need more precise considerations on the global properties of (H).