

80. On the Singularities of the Riemann Functions of Mixed Problems for the Wave Equation in Plane-Stratified Media. II

By Mutsuhide MATSUMURA

Faculty of Science, Tokyo University of Education

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In this note we shall continue our study of the Riemann function of the mixed problem (1)–(5) in the previous note [1].

3. Expression of the secondary Riemann function $F(x, y)$ ($=F_i(x, y)$ in Ω_i). The $F_1(x, y)$ and $F_2(x, y)$ are given by the following formulas.

Case $0 < y_n < h$.

$$\begin{aligned}
 (2\pi)^n F_1(x, y) &= \int_{S_m} \exp i\{\langle x' - y', \zeta' \rangle + (x_n - h)\lambda_1^+ - y_n \xi_n\} Q(\zeta) \begin{vmatrix} B_1(-\lambda_1^+) & C_1(\lambda_2^+) \\ B_2(-\lambda_1^+) & C_2(\lambda_2^+) \end{vmatrix} \\
 &\quad \times d\zeta / R(\zeta') P_1(\zeta) \\
 &+ \int_{S_m} -\exp i\{\langle x' - y', \zeta' \rangle - (x_n - h)\lambda_1^+ - y_n \xi_n\} Q(\zeta) \begin{vmatrix} B_1(\lambda_1^+) & C_1(\lambda_2^+) \\ B_2(\lambda_1^+) & C_2(\lambda_2^+) \end{vmatrix} \\
 &\quad \times d\zeta / R(\zeta') P_1(\zeta) \\
 (14) \quad &+ \int_{S_m} -\exp i\{\langle x' - y', \zeta' \rangle + x_n \lambda_1^+ + (h - y_n) \xi_n\} Q(-\lambda_1^+) \begin{vmatrix} B_1(\zeta) & C_1(\lambda_2^+) \\ B_2(\zeta) & C_2(\lambda_2^+) \end{vmatrix} \\
 &\quad \times d\zeta / R(\zeta') P_1(\zeta) \\
 &+ \int_{S_m} \exp i\{\langle x' - y', \zeta' \rangle - x_n \lambda_1^+ + (h - y_n) \xi_n\} Q(\lambda_1^+) \begin{vmatrix} B_1(\zeta) & C_1(\lambda_2^+) \\ B_2(\zeta) & C_2(\lambda_2^+) \end{vmatrix} \\
 &\quad \times d\zeta / R(\zeta') P_1(\zeta) \\
 &\equiv F_{1,1}(x, y) + F_{1,2}(x, y) + F_{1,3}(x, y) + F_{1,4}(x, y), \quad \text{respectively.}
 \end{aligned}$$

$$\begin{aligned}
 (2\pi)^n F_2(x, y) &= \int_{S_m} \exp i\{\langle x' - y', \zeta' \rangle + (x_n - h)\lambda_2^+ - y_n \xi_n\} Q(\zeta) \begin{vmatrix} B_1(\lambda_1^+) & B_1(-\lambda_1^+) \\ B_2(\lambda_1^+) & B_2(-\lambda_1^+) \end{vmatrix} \\
 &\quad \times d\zeta / R(\zeta') P_1(\zeta) \\
 (15) \quad &+ \int_{S_m} \exp i\{\langle x' - y', \zeta' \rangle + (x_n - h)\lambda_2^+ + (h - y_n) \xi_n\} \\
 &\quad \times \left[\exp \{-ih\lambda_1^+\} Q(\lambda_1^+) \begin{vmatrix} B_1(-\lambda_1^+) & B_1(\zeta) \\ B_2(-\lambda_1^+) & B_2(\zeta) \end{vmatrix} \right. \\
 &\quad \left. - \exp \{ih\lambda_1^+\} Q(-\lambda_1^+) \begin{vmatrix} B_1(\lambda_1^+) & B_1(\zeta) \\ B_2(\lambda_1^+) & B_2(\zeta) \end{vmatrix} \right] d\zeta / R(\zeta') P_1(\zeta) \\
 &\equiv F_{2,1}(x, y) + F_{2,2}(x, y) \quad \text{respectively.}
 \end{aligned}$$

Case $y_n > h$.