94. *Prime Closed Geodesics on Pinched Spheres**

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In this article we generalize the notion of type number for an abstract variation problem and count the number of prime closed geodesics on a pinched sphere (Theorem 3.2, 3.3).

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1. Definition 1.1. Let $(X, \varphi; f)$ be a triple of a topological space X, a continuous function φ on X such that $\varphi \geq 0$ and a continuous function φ on X such that $\varphi \geq 0$ and a continuous map f into itself. The triple $(X, \varphi; f)$ is a (abstract) variation problem (over a field k) if X, φ, and f satisfies the following:

i) $\varphi(f(x)) \leq \varphi(x)$ for any $x \in X$.

ii) $\varphi(f(x)) = \varphi(x)$ implies $f(x) = x$.

iii) the homomorphism f_* induced by f on $H_*(X; k)$ is the identity.

A point $x \in X$ for which $f(x) = x$ is said to be a critical point of $(X, \varphi; f)$ and the totality of the critical point is denoted by γ_f.

Definition 1.2. Let $(X, \varphi; f)$ be a variation problem. A norm $|A|$ of A is defined by the following, for any compact set A in X

$$|A| = \sup \left\{ \lim_{n \to \infty} \varphi(f^n(x)) : x \in A \right\}.$$

Then the triple $(X, \varphi; f)$ is said to have a norm if the norm above satifies the following:

iv) for any compact set A and for any neighborhood U of $\gamma_f \cap \varphi^{-1}$ $(|A|)$, there is an integer N such that $n \geq N$ implies $f^n(A) \subset U$ $\cup \varphi^{-1}([0, |A|))$.

Definition 1.3. *A* variation problem $(X, \varphi; f)$ is said to be discrete if

i) the set $(\gamma_f \cap \varphi^{-1}(a))'$ is discrete for any real number $a \geq 0$, where $(*)'$ is the derived set of $(*)$.

ii) $\varphi(\gamma_f)$ is discrete in real number.

Definition 1.4. Let X be a topological space and φ be a continuous function on X. Then an n-th type number $T_n(x; X, \varphi)$ of x is defined by the following:

$$T_n(x; X, \varphi) = \varprojlim H_n(U, U^-; k)$$

*) Dedicated to Professor Ryoji Shizuma on his 60th birthday.