93. *Finiteness Theorem for Holonomic Systems of Micro-differential Equations*

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It is known that the solution space of a holonomic system (=maximally overdetermined system) of linear *differential* equations enjoys a nice finiteness property (Kashiwara [2]). This result naturally raises an interesting question whether analogous results hold for holonomic systems of micro-differential equations (=pseudo-differential equations.) Of course, we should talk about the microfunction solutions in this case and this makes the situations complicated.

However, we can overcome the difficulties by making use of a recent result on the boundary value problem for elliptic systems (Kashiwara-Kawai [4]) on one hand and the concrete representation of the action of micro-differential operators on microfunctions (Kashiwara-Kawai [3] and Bony-Schapira [1]) on the other hand.

Our result is the following

Theorem. *Let M be a real analytic manifold, $\mathcal{C}$ the sheaf of microfunctions and $\mathcal{E}$ the sheaf of micro-differential operators. Let $\mathcal{M}$ be a holonomic system of micro-differential equations defined in a neighborhood of a point p of the pure imaginary cotangent bundle $\sqrt{-1}T^*M$. Then, the dimension of the vector space $\mathcal{E}xt^j_{\mathcal{E}}(\mathcal{M}, \mathcal{C})_p$ is finite for any j.*

We can prove this theorem in the following manner.

(I) Define a real hypersurface S in $\boldsymbol{C}^{n+1}$ by $\{(t, z) \in \boldsymbol{C}^{n+1}; \mathrm{Re}\, t = |z|^2\}$. Set $\Omega = \{(t, z) \in \boldsymbol{C}^{n+1}; \mathrm{Re}\, t > |z|^2\}$. We define $\mathcal{C}'$ by the inductive limit of $\mathcal{O}(U \cap \Omega)/\mathcal{O}(U)$, where U runs over a fundamental system of neighborhoods of $(t, z) = (0, 0)$. Then we can find an isomorphism between $\mathcal{E}_{M,p}$ and $\mathcal{E}_{\boldsymbol{C}^{n+1},(0,0;-dt)}$ and an isomorphism between $\mathcal{C}_{M,p}$ and $\mathcal{C}'$ so that the action of $\mathcal{E}_{M,p}$ on $\mathcal{C}_{M,p}$ is compatible with that of $\mathcal{E}_{\boldsymbol{C}^{n+1},(0,0;-dt)}$ on $\mathcal{C}'$. (Kashiwara-Kawai [3] § 2.1.)

Further, we can choose these isomorphisms so that the characteristic variety Λ of the $\mathcal{E}_{\boldsymbol{C}^{n+1},(0,0;-dt)}$-module $\mathcal{M}'$ corresponding to $\mathcal{M}$ is finite over $\boldsymbol{C}^{n+1}$, since the characteristic variety of $\mathcal{M}$ is Lagrangian.

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