112. On Nonlinear Evolution Equations with a Difference Term of Subdifferentials

By Yoshiharu KOI*) and Jiro WATANABE**)

(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 12, 1976)

1. Introduction. Let φ be a lower semicontinuous convex function on a real Hilbert space H into $(-\infty, \infty]$ with $\varphi \not\equiv \infty$. We define the subdifferential $\partial\varphi$ of φ by $\partial\varphi(v)=\{w \in H ; \varphi(u) \geqq \varphi(v)+(u-v, w)$ for all $u \in H\}$ for each $v \in H$. We set $D(\varphi)=\{v \in H ; \varphi(v)<\infty\}$ and $D(\partial\varphi)=\{v \in H ; \partial\varphi(v) \neq \phi\}$. For each $v \in D(\partial\varphi)$, $\partial\varphi^0(v)$ denotes the uniquely determined element of least norm in $\partial\varphi(v)$. Let ψ be another lower semicontinuous convex function on H into $(-\infty, \infty]$ with $\psi \not\equiv \infty$. We write $J(v)=\varphi(v)-\psi(v)$ for all $v \in D(\varphi)$.

Let us consider the nonlinear evolution equation

(1) $$du(t)/dt+\partial\varphi(u(t))-\partial\psi(u(t)) \ni f(t), \qquad t>0$$

with the initial condition

(2) $$u(0)=a.$$

We assume that φ and ψ satisfy the following conditions (I) and (II).

(I) $D(\varphi) \subseteqq D(\partial\psi)$.

(II) *If B is a bounded subset of $D(\varphi)$ such that J is bounded from above on B, then B is relatively compact in H and $\partial\psi^0$ is bounded on B.*

In this paper we will show that conditions (I) and (II) are sufficient for the local existence of solutions of the initial value problem (1), (2). We will also prove extension theorems for solutions. Recently M. Otani dealt with the initial value problem (1), (2) under certain assumptions which are sufficient for the global existence of solutions. Otani's conditions on φ and ψ are different from ours.

In another paper we will discuss applications to the initial boundary value problems for nonlinear parabolic equations treated by Tsutsumi [2].

2. Local existence theorem. Let I be an interval in $(-\infty, \infty)$. We denote by $L^2_{\mathrm{loc}}(I ; H)$ the space of all H-valued strongly measurable functions g on I such that

$$\int_K \|g(t)\|^2 dt<\infty$$

for all compact interval K in I. When $-\infty<r<s<\infty$, we write $L^2(r, s ; H)$ instead of $L^2_{\mathrm{loc}}([r, s] ; H)$.

Definition. Let u be an H-valued continuous function on $[0, S)$

*) Saitama Prefectural Toyooka Senior High School.
**) The University of Electro-Communications.