148. On a Theorem of Ph. Bénilan Concerning Semigroups Systems

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Let X be a real Banach space. By the *duality map* of X into X^*, the dual space of X, we mean the *multivalued mapping* F of X into X^* defined by $Fx=\{f\in X^*;\langle x,f\rangle=\|x\|^2=\|f\|^2\}$. The *tangent functional* $\tau(x,y)$ on $X\times X$ is defined by $\tau(x,y)=\lim_{t\downarrow 0} t^{-1}(\|x+ty\|-\|x\|)$ for $x,y\in X$, and it is known that $\tau(x,y)$ satisfies the following conditions: (a) $\tau(x,y)\leqq\|y\|$, (b) $\tau(x,y_1+y_2)\leqq\tau(x,y_1)+\tau(x,y_2)$, (c) $\tau(x,ay)=a\cdot\tau(x,y)$ for $a\geqq 0$, (d) $-\tau(x,-y)\leqq\tau(x,y)$, (e) $\tau(x,ax)=a\|x\|$ for real a, (f) $\|x\|\cdot\tau(x,y)=\sup_{f\in Fx}\langle y,f\rangle$. By a *semigroups system* on a closed set $D\subseteqq X$, we mean a family $\{S_y(t);t\geqq 0,y\in X\}$ of operators from D into itself satisfying

(1) $S_y(0)=I$ (the identity), $S_y(t+s)=S_y(t)S_y(s)$,

(2) $\lim_{t\downarrow 0} S_y(t)x=x$ for $x\in D$,

(3) $\|S_{y_1}(t)x_1-S_{y_2}(t)x_2\|\leqq\|S_{y_1}(s)x_1-S_{y_2}(s)x_2\|+\int_s^t\tau(S_{y_1}(\sigma)x_1-S_{y_2}(\sigma)x_2,y_1-y_2)d\sigma$ for $t\geqq s\geqq 0$ and $x_1,x_2\in D$ with $y_1,y_2\in X$.

A multivalued operator A defined on $D(A)\subseteqq X$ with values in X is called *accretive* if $\tau(x_1-x_2,y_1-y_2)\geqq 0$ for $y_i\in Ax_i$ $(i=1,2)$, and an accretive operator A is called *m-accretive* if the range $R(I+A)=\{x+y;y\in Ax,x\in D(A)\}=X$. In this note, we shall discuss the relation between semigroups systems and a family of m-accretive operators. We firstly prove the following

Theorem I. *If A is an m-accretive operator, then the operator $A-y(D(A)\ni x\to Ax-y)$ is also m-accretive and there exists a semigroups system $\{S_y(t);t\geqq 0,y\in X\}$ on the closure $\overline{D(A)}$ of $D(A)$ such that for each $x\in\overline{D(A)}$ we have $S_y(t)x=\lim_{\lambda\downarrow 0}(I+\lambda(A-y))^{-[t/\lambda]}\cdot x$ uniformly in t on every bounded interval of $[0,\infty)$.*

Proof. The proof of (1) and (2) is given by the Crandall-Liggett theorem and (3) is shown in a slightly different form by Bénilan (Thèse, Orsay (1972)). To give a straightforward proof of (3), we shall prepare the following inequality (suggested by I. Miyadera):

$$(3)'\qquad \|S_y(t)x-x_0\|\leqq\|S_y(s)x-x_0\|+\int_s^t\tau(S_y(\sigma)x-x_0,y-y_0)d\sigma$$

for $x\in D(A)$, $y_0\in Ax_0$ and $t\geqq s\geqq 0$.