145. On the Distribution of Zeros of Dirichlet's L-Function on the Line $\sigma=1/2$

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§1. **Introduction.** Results on the distribution of zeros of Dirichlet's L-function on the line $\sigma=1/2$ have been proved by analogous method in case of Riemann's ζ-function. For example, Hardy proved in 1914 that there exist infinitely many zeros of Riemann's ζ-function on the critical line and later Hardy and Littlewood proved that

$$N_0(T)>KT$$

for some absolute constant K and then these results were easily extended in case of $L(s,\chi)$. (See Süetuna [8] Chap. III.) In 1942, A. Selberg proved that

$$N_0(T)>cT\log T$$

for some constant c and this method was also applicable to $L(s,\chi)$. Recently N. Levinson gave a different proof of Selberg's result with $c=1/3$.

In this note we shall show that the essential idea of Levinson is also applicable to the case of $L(s,\chi)$ in order to prove the fundamental properties of $L(s,\chi)$. Details of the calculation will appear elsewhere.

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§2. **Fundamental properties of $L(s,\chi)$.** Throughout this note, χ denote a primitive character with mod q and T is a sufficiently large number. We use the following notations;

$$a=\frac{1}{2}(1-\chi(-1)) \tag{2.1}$$

$$h(s)=h(s,\chi)=\left(\frac{\pi}{q}\right)^{-(s+a)/2}\Gamma\left(\frac{s+a}{2}\right) \tag{2.2}$$

$$\varepsilon(\chi)=\frac{(-i)^a}{q^{1/2}}\sum_{m=1}^{q}\chi(m)e^{2\pi im/q} \tag{2.3}$$

$$f'(s)=h'(s)/h(s). \tag{2.4}$$

As is well known, we have

$$|\varepsilon(\chi)|=1.$$

We can choose a complex number α with

$$\bar{\alpha}=\alpha^{-1}$$