123. *On Zero Points of a Meromorphic Function of Finite Order.*

By Satoru TAKENAKA.

Shiomi Institute, Ôsaka.

(Rec. Sep. 5, 1926. Comm. by M. FUJIWARA, M.I.A., Oct. 12, 1926.)

The second theorem proved by Mr. TSUJI in his paper: "On the zero points of a bounded analytic function"[1], concerning the roots of a transcendental integral function of finite order, can be extended to the meromorphic function of finite order.

Let $f(z)$ be a meromorphic function of finite order[2], regular at $z=0$.

Let $\lambda_1(x), \lambda_2(x), \cdots$ denote the roots of $f(z)=x$ in ascending order of absolute values,

$$|\lambda_\nu(x)| = r_\nu(x), \quad (\nu=1, 2, \cdots)$$

and $n(r; x)$ the number of the roots of $f(z)=x$ in $|z|<r$ and especially

$$\lambda_\nu(\infty) = \lambda_\nu, \quad r_\nu(\infty) = r_\nu, \quad (\nu=1, 2, \cdots) \quad \text{and} \quad n(r; \infty) = n(r).$$

Then by similar method as in my paper "On the power series etc.", Japanese Journal of Math., 2 (1925), 88, we can prove the following theorem:

Theorem I. *Consider a set of functions $\{f(z)\}$, which have the following properties:*

(i) *$f(z)$ is meromorphic in $|z| \leqq r$,*

(ii) *$f(0)=1$,*

(iii) *$f(z)$ has m roots and $n(r)$ poles in $|z|<r$.*

Then among such functions the unique one corresponding to the minimum of the integral

$$\frac{1}{2\pi}\int_0^{2\pi} \overset{+}{\log} |f(r e^{i\theta})| \, d\theta$$

1) These Proceedings, 2 (1926), 248.

2) The definition of order here used is due to R. NEVANLINNA. See R. NEVANLINNA, Zur Theorie der meromorphen Funktionen, Acta Math., 46 (1925), 23, or VALIRON, Fonctions entières et fonctions méromorphes d'une variable, Paris (1925), 37–38.