121. Note on the Conformal Representation.

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Let $V_\nu(z)$, $(\nu=0, 1, 2,\cdots)$ be a set of regular analytic functions, which form a complete system of normalized orthogonal functions on a simply closed analytic curve C of length l:

$$\frac{1}{l}\int_c V_\mu(z)\overline{V_\nu(z)}\,ds \begin{cases} = 0 & \text{for } \mu \neq \nu, \\ = 1 & \text{for } \mu = \nu. \end{cases}$$

Then the series

$$\sum_{\nu=0}^{\infty} V_\nu(z)\,\overline{V_\nu(a)}, \quad (a \text{ in } C)$$

is convergent absolutely and uniformly in the closed region interior to C, and represents a definite function $K(z, a)$ dependent only on the curve C.

Now let $\{f(z)\}$ be a set of functions, regular and analytic in C, such that

$$\frac{1}{l}\int_c |f(z)|^p\,ds \leqq 1, \quad (p>0).$$

Of these functions that which makes $|f(\zeta)|$ (ζ in C) a maximum is

$$f^*(z) = \varepsilon_1 \left\{\frac{K(z, \zeta)^2}{K(\zeta, \zeta)}\right\}^{\frac{1}{p}}, \qquad (|\varepsilon_1| = 1)^{1)}$$

This problem may also be solved by the conformal transformation.

Let $x=\chi(z, a)$ be the equation by which the interior of C is transformed conformally into the interior of the unit circle about the origin of the x-plane, the point a corresponding to the origin, and let $z=\omega(x, a)$ be the inverse representation.

1) S. TAKENAKA, General mean modulus of analytic functions, Tôhoku Math. Journal, 27 (1926).