31. *On the Power Series Whose Initial Coefficients are Given.*

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Let C_ν, $(\nu=0,1,\cdots,n)$ be given constants not all zero (without any loss of generality let us suppose that $C_0\neq 0$); and consider a set $\{f(z)\}$ of functions, regular and analytic for $|z|<1$, and such that

$$f(z)\equiv\sum_{\nu=0}^{n}C_\nu z^\nu,\quad(\text{mod. } z^{n+1}).$$

Of these functions that which makes the integral

$$I(f)=\frac{1}{2\pi}\int_{|z|=1}|f(z)|\cdot|dz|$$

minimum is a rational integral function $f^*(z)$ of a degree not exceeding $2n$. This result has been proved by F. RIESZ[1].

In this note I will give the inferior limit of $I(f)$ and the true expression of $f^*(z)$ when C_ν $(\nu=0,1,\cdots,n)$ satisfy certain conditions.

Let $\varphi(z)$ be an arbitrary regular function of the form

$$\varphi(z)=\sum_{\nu=0}^{\infty}a_\nu z^\nu,\qquad(|z|<1),$$

under the condition that

$$|\varphi(z)|\leqq M\quad\text{for } |z|<1.$$

Then it can easily be seen that

$$\left|\sum_{\nu=0}^{n}C_\nu a_{n-\nu}\right|\leqq\frac{M}{2\pi}\int_{|z|=1}|f(z)|\cdot|dz|. \tag{1}$$

Now put

$$(x_0+x_1z+\cdots+x_nz^n)^2\equiv\sum_{\nu=0}^{n}C_\nu z^\nu,\ (\text{mod. } z^{n+1}),$$

1) F. RIESZ, Ueber Potenzreihen mit vorgeschriebenen Anfangsgliedern, Acta Math., 42 (1920), 145.