103 On a Problem Proposed by Hardy and Littlewood.

(*The Fourth Report on the Order of Linear Form.*)

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1. We consider the function $\varphi_{\alpha,\beta}(t)$, which is the minimum absolute value of $t(\alpha x\text{-}y\text{-}\beta)$ for the integral values of x and y, where $|x| < t$. In the former reports I treated mainly the problem of finding the inferior limit of this function, which may be considered as an extension of a problem solved by Minkowski. On the other hand, Hardy and Littlewood have proposed in the paper "On some Problem of diophantine Approximation"[1] to determine the superior limit of this function, and Khintchine has proved that, if

$$\limsup \varphi_{\alpha,\beta}(t) = \infty \tag{1}$$

then the denominators (a_n) of the simple continued fraction for α can not be limited, and conversely, if they are not limited, then we can choose β, such that (1) subsists.[2] I wish to apply the same idea as in my former reports to this problem.

2. Let us consider on the xy-plane a system of lattice points, corresponding to the integral values of x and y, and the line L: $\alpha x - y - \beta = 0$ and Y: $x=0$, whose intersection is supposed to be M. First we construct a parallelogramm, whose sides are parallel to L and Y and whose center is M and which contains no lattice point in it. We translate the upper and the lower sides (which are parallel to L) away from L till a lattice point $P_{n(k)}$ comes on one of these sides and again translate the left and the right sides (which are parallel to Y) away from Y till a lattice point $P_{n(k+1)}$ comes on one of these sides. Next we draw a parallel line to L through $P_{n(k+1)}$ and taking this line as the upper or lower side we construct the parallelogramm in a similar manner as above, which contains no lattice point in it, but the lattice point $P_{n(k+2)}$ on one of the right or left side, and so on. Thus we have a series of parallelogramms

$$S_{n(k)},\ S_{n(k+1)},\ S_{n(k+2)},\ \dots\dots,$$

and of the points

$$P_{n(k)},\ P_{n(k+1)},\ P_{n(k+2)},\ \dots\dots.$$

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1) Acta Mathematica **37** (1914), pp. 155–191.

2) Über die angenäherte Auflösung linearer Gleichungen in ganzen Zahlen, Recueil de Mathématiques de Moscou, **32** (1924), pp. 203–219.