102. *On the Theory of Surfaces in the Affine Space.*

By Buchin SU.
Mathematical Institute, Tohoku Imperial University, Sendai.

(Rec. June 20, 1928. Comm. by M. FUJIWARA, M.I.A., July 12, 1928.)

In connection to an interesting investigation on closed convex surfaces due to Prof. T. Kubota[1] the main object of this note is to treat the surfaces, called *affine moulding surfaces*, having ∞^1 system of curves which lie on parallel planes and are at the same time curves of contact of the enveloping cones with the surface. A special class of the affine moulding surfaces are defined as *affine surface of revolution*. For the latter we show that our definition is equivalent to that considered by Dr. Süss.[2]

1. The equation to the affine moulding surface may be easily deduced. In fact, take the curves on parallel planes ("parallel curves" say) and its conjugate system (" meridian curves " say) as parametric curves v, u; and let $(\xi(v), \eta(v), \zeta(v))$ be the curve (Γ-curve say) on which the vertices of the enveloping cones along v=const. lie. Then the surface is given by the equations:

$$x=\exp\left(-\int\phi\, dv\right)\left(\int\xi\, \phi\exp\left(\int\phi\, dv\right)dv+U_1\right),$$
$$y=\exp\left(-\int\phi\, dv\right)\left(\int\eta\, \phi\exp\left(\int\phi\, dv\right)dv+U_2\right),$$
$$z=\exp\left(-\int\phi\, dv\right)\left(\int\zeta\, \phi\exp\left(\int\phi\, dv\right)dv+U_3\right),$$

where $\phi=(a\xi+b\eta+c\zeta-v)^{-1}$, a, b, c being the (constant) direction cosines of the parallel planes; and U_1, U_2, U_3 are functions of u alone satisfying the relation

$$a\,U_1+b\,U_2+c\,U_3=0\,.$$

In what follows we will denote the surface in consideration by $[a, \xi, U]$.

Putting

$$T=\begin{pmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \\ \alpha_3 & \beta_3 & \gamma_3 \end{pmatrix}, \qquad \overline{T}=\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{pmatrix},$$

we can prove the formula

1) T. Kubota, On the theory of closed convex surface, Proc. London Math. Soc. Ser. 2, **14** (1914); or Science Reports, Tohoku Imp. Univ. Ser. 1, **3** (1914).

2) W. Süss, Ein affingeometrische Gegenstück zu den Rotationsflächen, Math. Annalen, **98** (1928).