101. Differential Geometry of Conics in the Projective Space of Three Dimensions.

III. *Differential invariant forms in the theory of a two-parameter family of conics (second report).*

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4. *Normalization of* $\mathfrak{l}$. A two-parameter family of conics in the projective space of three dimentions can be represented by the equations in the parametric form

$$\mathfrak{a}=\mathfrak{a}(u^1, u^2),\quad \mathfrak{l}=\mathfrak{l}(u^1, u^2), \tag{19}$$

when we adopt the coordinate system of a conic in space, introduced in one of my previous papers[1]. For the system $\mathfrak{a}$ we have already completely discussed in the first report, and we may use the differential forms and the results in that report, because the present theory can be got by a proper combination of those of a conic-family in a plane (theory of $\mathfrak{a}$) and of a surface in space (theory for $\mathfrak{l}$). We must, therefore, introduce other differential invariant forms connected with the family, besides those introduced in the first report.

Put

$$H=h_{ij}du^i du^j=\frac{1}{\sqrt{G}}\,|\,\mathfrak{l}\,\mathfrak{l}_1\,\mathfrak{l}_2\,\mathfrak{l}_{ij}\,|\,du^i du^j, \tag{20}$$

which is an invariant differential form, where

$$G=g_{11}g_{22}=g_{12}^2$$

and $\mathfrak{l}_i$, $\mathfrak{l}_{ij}$ are the first and the second covariant derivatives of $\mathfrak{l}$ with respect to the form $g_{ij}du^i du^j$. Moreover we introduce the quantities h_{ij} such that

$$h^{ij}h_{ik}=\delta_k^j \tag{21}$$

and normalize the coordinates $\mathfrak{l}$ so that they satisfy the relation

$$h^{ij}g_{ij}=1, \tag{22}$$

since h^{ij} is multiplied by ρ^{-4} corresponding to a change of proportional facter: $\rho\mathfrak{l}$.

5. *Another differential form.* Consider the differential form of the third order

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1) Differential geometry of conics in the projective space of three dimensions, I. Fundamental theorem in the theory of a one-parameter family of conics, these Proceedings **4**, 255–258.