100. Differential Geometry of Conics in the Projective Space of Three Dimensions.

II. Differential invariant forms in the theory of a two-parameter family of conics (first report).

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In my previous paper[1] I have built the theory of a one-parameter family of conics in the projective space of three dimensions. In this little note I will discuss the theory of a two-parameter family of conics in the projective space of three dimensions, as a continuation of that paper. This is done by some modifications of my theory of a m-parameter family of hypersurfaces of the second order in the projective space of n dimensions[2] and of Fubini's surface-theory in the projective space[3]. In this first report I will discuss, as a preliminary, the theory of a two-parameter family of conics in the plane, modifying my theory in the n-dimensional space[4].

1. *The differential forms.* A two-parameter family of conics in the plane can be represented by the equations in parametric form

$$\mathfrak{a}=\mathfrak{a}(u^1, u^2),$$

where u^1 and u^2 are two parameters, when we adopt the coordinate-system $\mathfrak{a}$ of the conic in the plane, which has been introduced in my previous paper[5]. We assume $\mathfrak{a}$ so normalized that

$$(\mathfrak{a}, \mathfrak{a}, \mathfrak{a})=1,$$

i.e.
$$\mathfrak{a}=(\bar{\mathfrak{a}}, \bar{\mathfrak{a}}, \bar{\mathfrak{a}})^{-\frac{1}{3}}\bar{\mathfrak{a}}.$$

Let us consider the differential forms:

$$g_{ij}du^i du^j=2(\mathfrak{a}_i, \mathfrak{a}_j, \mathfrak{a})du^i du^j, \tag{1}$$

$$a_{ijk}du^i du^j du^k=(\mathfrak{a}_i, \mathfrak{a}_j, \mathfrak{a}_k)du^i du^j du^k, \tag{2}$$

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1) Differential geometry of conics in the projective space of three dimensions, I. Fundamental theorem in the theory of a one-parameter family of conics, these Proceedings **4** (1928), 255–258.

2) See my paper, Fundamental forms in the projective differential geometry of m-parametric families of hypersurfaces of the second order in the n-dimensional space, these Proceedings, **3** (1927), 310–314, and Ueber projektive Differentialgeometrie V, which will be published in the Tohoku Mathematical Journal.

3) See G. Fubini-E. Čech, Geometria proiettiva differenziale, I and II, Bologna, 1926–27.

4) *loc. cit.*

5) *loc. cit.*