PAPERS COMMUNICATED

116. *On the Convergency of the Series Summable (C, r).*

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1. In the Tôhoku Mathematical Journal, 33 (1930) Mr. Izumi treated of the condition for the convergency of the series summable (C, r), and gave a simple proof for Hardy-Landau's theorem in the following generalized form:

A. *If the series is summable (C, r) $(r>0)$, and*

$$\liminf (s_m - s_n) \geqq 0, \quad \textit{for} \quad m>n,\ n\to\infty,\ \frac{m}{n}\to 1,$$

where s_n denotes the sum of the first $n+1$ terms of the series, then the series is convergent.

In the proof for the case $r=2$, he started from

$$V_{mn}=S_m^{(2)}-S_n^{(2)}-(m-n)S_n^{(1)}-\frac{1}{2!}(m-n)(m-n+1)S_n^{(0)},$$

$$W_{mn}=S_m^{(2)}-S_n^{(2)}-(m-n)S_m^{(1)}+\frac{1}{2!}(m-n)(m-n-1)S_n^{(0)}.$$

If we take instead of V_{mn}, W_{mn}

$$U_{mn}=S_m^{(2)}-2S_\mu^{(2)}+S_n^{(2)}-\left(\frac{m-n}{2}\right)^2 S_n^{(2)}, \quad \left(\mu=\frac{m+n}{2}\right)$$

where $m-n$ is an even number, the proof will be much simpler.

For the general case, where r denotes any positive integer, we have to put

$$U_{mn}^{(r)}=S_{n+rl}^{(r)}-\binom{r}{1}S_{n+(r-1)l}^{(r)}+\binom{r}{2}S_{n+(r-2)l}^{(r)}-\cdots\cdots+(-1)^r S_n^{(r)}-l^r S_n^{(0)},$$
$$(m=n+rl).$$

2. In the following lines I wish to give the proof of the theorem in more general form.

Suppose that the series $\sum a_n$ is summable (C, r) to the sum s, where r denotes any positive integer. Then it is well known that

$$\lim_{n\to\infty} S_n^{(r)}\Big/\binom{n+r}{r}=s,$$

where $\quad S_n^{(\rho)}=S_0^{(\rho-1)}+S_1^{(\rho-1)}+S_2^{(\rho-1)}+\cdots\cdots+S_n^{(\rho-1)} \quad (\rho=1, 2, \ldots\ldots, r),$