163. A New Concept of Integrals.

By Shin-ichi IZUMI.

Mathematical Institute, Tohoku Imperial University, Sendai.

(Comm. by M. FUJIWARA, M.I.A., Dec. 12, 1933.)

The object of this paper is to define the integral, which is more general than the Ridder's integral, and then than those of Danjoy and Burkill.

1. Let $f(x)$ be defined in an interval (a, b). If we can find a set E, such that

1°. x is the point of exterior density of E,

2°. $$\lim_{\xi(\varepsilon E)\to x} \frac{f(\xi)-f(x)}{\xi - x} \tag{1}$$

exists and is finite, where the limit is taken such that ξ tends to x, lying in E, then $f(x)$ is called to be *approximately differentiable* at x, and the value (1) is called the *approximate derivative* at x and is denoted by $ADf(x)$.

Theorem 1. If $f(x)$ is measurable and is approximately differentiable at x, then there is a measurable set E, such that

1°. x is the point of density of E,

2°. $\lim_{\xi(\varepsilon E)\to x} \frac{f(\xi)-f(x)}{\xi - x}$ exists and is equal to $ADf(x)$.

2. If we can find a set E, such that

1°. the inferior interior density of E at x is $\geqq \tau (>0)$,

2°. $$\lim_{\xi(\varepsilon E)\to x} \frac{f(\xi)-f(x)}{\xi - x} \tag{2}$$

exists and is finite, then $f(x)$ is called to be (τ)-*approximately differentiable* at x. The value (2) is called the (τ)-*approximate derivative* of $f(x)$ at x, and is denoted by $\underset{\tau}{A}Df(x)$. And put $\underset{1}{A}Df(x) = A^*Df(x)$.

Next, let E be a set such that the inferior interior density on the right hand of E at x is $\geqq \tau (>0)$.

Put $$a_E = \overline{\lim_{\xi(\varepsilon E)\to x}} \frac{f(\xi)-f(x)}{\xi - x}.$$

The lower bound of a_E is defined to be the *upper* (τ)-*approximate derivative on the right hand* of $f(x)$ at x, and denoted by $\underset{\tau}{A}D^+f(x)$. When