15. *On the Convergence Factor of the Fourier-Denjoy Series.*

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Hardy has shown that $(\log n)^{-1}$ is a convergence factor of the Fourier-Lebesgue series. The object of this paper is to show that n^{-1} is a convergence factor of the Fourier-Denjoy series, and to construct an example such that $n^{-\delta}$ $(0<\delta<1)$ is not the convergence factor of the Fourier-Denjoy series.

1. Let $f(x)$ be a function, integrable in Denjoy-Perron's sense and periodic, with period 2π. And let

$$f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx)\,. \tag{1·1}$$

Then we have

Theorem. n^{-1} *is a convergence factor of the Fourier-Denjoy series* (1·1). *In fact,*

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}\frac{a_n\cos nx+b_n\sin nx}{n} \tag{1·2}$$

converges almost everywhere.

In order to prove the theorem, we require the following

Lemma.[1] *The Fourier-Denjoy series* (1. 1) *is summable* $(C, 1+\delta)$ $(\delta>0)$ *almost everywhere.*

Put $$s_n(x)=\frac{a_0}{2}+\sum_{k=1}^{n}(a_k\cos kx+b_k\sin kx)\,,$$

$$\phi(t)=\frac{1}{2}\{f(x+t)+f(x-t)-2f(x)\}\,,$$

and $$\phi_1(t)=\int_0^t\phi(u)du\,.$$

Then $$\phi_1(t)=o(t)^{2)}$$

for almost all values of x in $(-\pi, \pi)$, and then

1) Priwalof: Rend. di Palermo, **41** (1916).
c.f. Bosanquet, Proc. London math. soc. 31.

2) Hobson: Theory of function, vol. I (1921), p. 642.