55. Some Intrinsic Derivations in a Generalized Space.

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In their very interesting papers H. V. Craig[1)] and J. L. Synge[2)] defined a Kawaguchi space and stated many intrinsic vectors and derivatives in this space. I shall introduce in this paper some another intrinsic derivations in the same space, which do not take place in the space of order one.

1. In a Kawaguchi space of order m and of dimensions n the length of a curve $x^i=x^i(t)$ $(i=1, 2, \ldots\ldots, n)$ is defined to be the invariant

$$s=\int_{t_0}^{t} F(t, x^{(0)i}, \ldots\ldots, x^{(m)i})\, dt\,,$$

where

$$x^{(a)i}=\frac{d^a x^i}{dt^a}\,, \qquad x^{(0)i}=x^i\,.$$

We shall adopt the notations

$$(1) \qquad F_{(\beta)i}=\frac{\partial F}{\partial x^{(\beta)i}}\,, \qquad F_{(\dot\beta)i}^{\cdot\cdot(a)}=\frac{d^a}{dt^a}\frac{\partial F}{\partial x^{(\beta)i}}\,,$$

then it was proved by Synge[2)] that

$$(2) \qquad \overset{a}{E}_i=\sum_{\beta=0}^{m}(-1)^\beta\binom{\beta}{a}F_{(\dot\beta)i}^{\cdot\cdot(\beta-a)} \qquad (a=0, 1, 2, \ldots\ldots, m)$$

are the components of a covariant vector, which we shall call the Synge vector of a-th kind. The Synge vector of zeroth kind is the Euler vector.

2. Let T_i be any covariant vector[3)] of order p, i. e. depended upon t, $x^{(0)i}, \ldots\ldots, x^{(p)i}$ and X^i a contravariant vector of any order, then we have the following covariant derivation along a curve for the vector X^i refered to the vector T_i.

Theorem. *The n quantities*

$$(\mathrm{I}) \qquad \overset{p-\rho}{D}_{ij}(T)X^j=\sum_{a=\rho}^{p}\binom{a}{\rho}T_{i(a)j}X^{j(a-\rho)} \qquad (\rho=1, 2, \ldots\ldots, p)$$

are the components of a covariant vector.

Proof. A point transformation $y^i=y^i(x^j)$ gives rise to the relations

$$(3) \qquad \frac{\partial y^{(a)i}}{\partial x^{(\beta)j}}=\binom{a}{\beta}P_j^{i(a-\beta)}\,,$$

where

$$P_j^i=\frac{\partial y^i}{\partial x^j}\,, \qquad Q_j^i=\frac{\partial x^i}{\partial y^j}\,.$$

1) H. V. Craig: On a generalized tangent vector, American Journal of Mathematics, **57** (1935), 456–462.

2) J. L. Synge: Some intrinsic and derived vectors in a Kawaguchi space, ibid., 679–691.

3) We can prove the analogous result, taking a contravariant vector T^i instead of the covariant vector T_i.