PAPERS COMMUNICATED

24. *An Application of the Fourier Transform to Almost Periodic Function.*

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Prof. B. Jessen has remarked that the theorems of R. Petersen and S. Takahashi on the formal differentiation and integration of the Fourier series of an almost periodic function may be considered as special cases of the following general theorem due to S. Bochner.[1]

Theorem. Let $K(x)$ denote a function of a real variable such that the integral

$$\int_{-\infty}^{\infty} |K(x)|\, dx$$

is convergent, and let $G(\lambda)$ denote its Fourier transform

$$G(\lambda)=\int_{-\infty}^{\infty} K(x)\, e^{-i\lambda x} dx\,.$$

Then if $f(t)$ is an almost periodic function of a real variable with the Fourier series

$$f(t)\sim \sum A_n e^{i\lambda_n t}\,,$$

the series

$$\sum G(\lambda_n) A_n e^{i\lambda_n t}$$

is also the Fourier series of an almost periodic function, namely of the function

$$g(t)=\int_{-\infty}^{\infty} f(-x+t)\, K(x)\, dx\,.$$

By use of this theorem, we can now obtain the following two theorems which correspond to the most general theorems on the formal differentiation and integration of the Fourier series of an almost periodic function.

Theorem 1. Let $f(t)$ be an almost periodic function of a real variable with the Fourier series

$$f(t)\sim \sum A_n e^{i\lambda_n t}\,.$$

Then the two series

$$\sum_{\lambda_n<0} |\lambda_n|^p A_n e^{\lambda_n s}\,, \qquad \sum_{\lambda_n>0} \lambda_n^p A_n e^{\lambda_n s} \qquad (s=\sigma+it)$$

where p is any positive number, are the Dirichlet series of two functions $f_1(s)$, $f_2(s)$ respectively almost periodic in $[0, +\infty)$ and in $(-\infty, 0]$.

1) Jessen, Remark on the theorems of R. Petersen and S. Takahashi, Matematisk Tidsskrift B (1935).