PAPERS COMMUNICATED

7. *On the Generalized Circles in the Conformally Connected Manifold.*

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As in Mr. K. Yano's paper[1)] in which the same problem is studied, take in the tangential space an $(n+2)-$ spherical "repère naturel" $[A_P]$ satisfying the following equations[2)]:

$$A_0^2=A_\infty^2=A_0A_i=A_\infty A_j=0\,,\quad A_0A_\infty=-1\,,\quad A_iA_j=G_{ij}=\frac{g_{ij}}{g^{\frac{1}{n}}}\,,\quad (1)$$
$$(i, j, k, \dots=1, 2, \dots, n)$$

the connection being defined by

$$dA_P=\omega_P^Q A_Q\,,\qquad (P, Q, R, \dots=0, 1, \dots, n, \infty) \qquad (2)$$

where

$$\omega_P^Q=\Pi_{Pk}^Q dx^k\,, \qquad (3)$$

$$\left.\begin{aligned}\Pi_{0k}^\infty=\Pi_{\infty k}^0=\Pi_{0k}^0=\Pi_{\infty k}^\infty=0\,,\quad \Pi_{0j}^i=\delta_j^i\,,\quad \Pi_{jk}^\infty=G_{jk}\,,\quad G_{ij}\Pi_{\infty k}^j=\Pi_{jk}^0\\ \Pi_{jk}^i=\frac{1}{2}G^{ih}(\partial_jG_{kh}+\partial_kG_{jh}-\partial_hG_{jk})\end{aligned}\right\} \qquad (4)$$

Then any curve $x^i(s)$ in the manifold can be developed into a curve in the tangential space at any point $x^i(s_0)$ on the curve by the formulae (2). Let us consider the curves whose developments are circles.

When we take two quantities a^P and b^P which are contragradient to A_P and satisfy the equations

$$\left.\begin{aligned}G_{PQ}a^Pa^Q=1\,,\qquad G_{PQ}a^Pb^Q=0\,,\qquad G_{PQ}b^Pb^Q=0\,,\\ a^\infty=0\,,\end{aligned}\right\} \qquad (5)$$

where

$$G_{PQ}=A_PA_Q\,,$$

then

$$\frac{1}{b^\infty}A_0+a^\alpha A_\alpha t+\frac{1}{2}b^PA_Pt^2 \qquad (\alpha=0, 1, 2, \dots, n) \qquad (6)$$

is an invariant and represents a circle in the tangential space. Because of (5), (6) becomes, when multiplied by b^∞,

$$\begin{aligned}A&=A_0+b^\infty a^\alpha A_\alpha+\frac{1}{2}b^\infty b^PA_Pt^2\\ &=\Big(1+G_{ij}a^ib^jt+\frac{1}{4}G_{ij}b^ib^jt^2\Big)A_0+\Big(b^\infty a^it+\frac{1}{2}b^\infty b^it^2\Big)A_i+\frac{1}{2}(b^\infty t)^2A_\infty\,. \qquad (7)\end{aligned}$$

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1) K. Yano: Sur les circonférences généralisées dans les espaces à connexion conforme, Proc. **14** (1938), 329-32.

2) K. Yano: Remarques relatives à la théorie des espaces à connexion conforme, Comptes Rendus, **206** (1938), 560-2.