66. Some Results in the Operator-Theoretical Treatment of the Markoff Process.

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Let us denote by $P(t,E)$ the transition probability that a point $t\in\Omega$ is transferred, by a simple Markoff process, into a Borel set E of Ω after the elapse of a unit-time. We have always $P(t,E)\geqq 0$ and $P(t,\Omega)=1$. We shall assume that $P(t,E)$ is completely additive for Borel sets E if t is fixed, and that $P(t,E)$ is Borel measurable in t if E is fixed. Then the probability $P^{(n)}(t,E)$ that t is transferred into E after the elapse of n unit-times is given recurrently by

$$P^{(n)}(t,E)=\int_{\Omega}P^{(n-1)}(t,ds)P(s,E),\ n=2,3,\dots;\ P^{(1)}(t,E)=P(t,E),$$

where the integration is of Radon-Stieltjes type.

Consider the complex Banach space $(\mathfrak{M})$ of all the complex-valued completely additive set functions $x(E)$ defined for all Borel sets E of Ω. For any $x(E)\in(\mathfrak{M})$, its norm is defined by $\|x\|=$ total variation of $|x(E)|$ on Ω. Then the integral operator

$$x\rightarrow T(x)=y:\quad y(E)=\int_{\Omega}x(dt)P(t,E)$$

is a bounded linear operation which maps $(\mathfrak{M})$ into itself and $\|T\|=1$. On the other hand, if we consider the complex Banach space (M^*) of all the complex-valued bounded measurable functions $x(t)$ defined on Ω, with $\|x\|=\underset{t\in\Omega}{\text{l.u.b.}}|x(t)|$ as its norm, then

$$x\rightarrow \bar{T}(x)=y:\quad y(t)=\int_{\Omega}P(t,ds)x(s)$$

is also a bounded linear operation which maps (M^*) into itself and $\|\bar{T}\|=1$.

Our main object is to investigate the asymptotic behaviour of $P^{(n)}(t,E)$ for large n. Since, as is easily seen, T^n and $\bar{T}^n$ are the integral operators defined by the kernel $P^{(n)}(t,E)$, our problem is transformed into the investigation of the iterations T^n and $\bar{T}^n$ of T and $\bar{T}$ respectively.

This investigation was carried out by K. Yosida.[1] Under the condition of N. Kryloff-N. Bogolioùboff:[2]

1) K. Yosida: Operator-theoretical treatment of the Markoff process, Proc. **14** (1938), 363-367.

2) N. Kryloff-N. Bogolioùboff: Sur les probabilités en chaîne, C. R. Paris, **204** (1937), 1386-1388.

N. Kryloff-N. Bogolioùboff: Les propriétés ergodiques des suites des probabilités en chaîne, C. R. Paris, **204** (1937), 1454-1455.