20. *Conformally Separable Quadratic Differential Forms.*

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1. Let us consider an n-dimensional Riemannian space whose first fundamental form is

$$ds^2 = g_{\mu\nu} du^\mu du^\nu, \qquad (\lambda, \mu, \nu, \dots = 1, 2, 3, \dots, n). \tag{1.1}$$

In this space, the equations $u^a = \text{const.}$ $(a, b, c, \dots = 1, 2, \dots, m; m < n)$ define a family of $(n-m)$-dimensional subspaces V_{n-m}, and the equations $u^i = \text{const.}$ $(h, i, j, \dots = m+1, m+2, \dots, n)$ a family of m-dimensional subspaces V_m in V_n.

It has been shown by E. Bompiani[1] that the necessary and sufficient condition that the subspaces V_{n-m} and the subspaces V_m orthogonal to V_{n-m} be totally geodesic in V_n is that

$$g_{ab} = f_{ab}(u^c), \qquad g_{jk} = f_{jk}(u^i), \qquad g_{ai} = 0, \tag{1.2}$$

and consequently the first fundamental form (1.1) may be written in the form

$$ds^2 = f_{ab}(u^c) du^a du^b + f_{jk}(u^i) du^j du^k. \tag{1.3}$$

In this case, the quadratic differential form (1.1) is said to be separable, and $f_{ab}(u^c)du^a du^b$ and $f_{jk}(u^i)du^j du^k$ are called the components of the separable quadratic differential form (1.1).

Recently, A. Fialkow[2] has proved that if the first fundamental form of an Einstein space of dimensionality $n > 3$ of mean curvature a is separable into two components whose dimensions exceed 1, then each component is also the first fundamental form of an Einstein space of mean curvature a.

In the present Note, we try to find the necessary and sufficient condition that the subspaces V_{n-m} and the subspaces V_m orthogonal to V_{n-m} be both totally umbilical in V_n, and to obtain a theorem corresponding to the theorem of A. Fialkow quoted above.

2. We assume that the subspaces V_{n-m} and the subspaces V_m orthogonal to V_{n-m} be both totally umbilical $(n-m, m \geqq 2)$ in V_n. The orthogonality between V_{n-m} and V_m gives us immediately

$$g_{ai} = 0, \qquad g^{ai} = 0. \tag{2.1}$$

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1) E. Bompiani, Spazi Riemanniani luoghi di varieta totalmente geodetiche, Rendiconti del Circolo Matematico di Palermo, **48** (1924) p. 124.

2) A. Fialkow, Totally geodesic Einstein spaces, Bulletin of the American Mathematical Society, **45** (1939) p. 423.