PAPERS COMMUNICATED

47. *Concircular Geometry I. Concircular Transformations.*

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§ 1. Let $C: u^\lambda(s)$ be a curve in a Riemannian space V_n whose fundamental quadratic form is

$$(1.1)\qquad ds^2 = g_{\mu\nu}du^\mu du^\nu\,, \qquad (\lambda, \mu, \nu, \dots = 1, 2, 3, \dots, n)\,.$$

Denoting the unit tangent, and unit normals of order $1, 2, \dots, n-1$ and the first, second, ... $(n-1)$-st curvatures of C by $\underset{1}{\xi}{}^\lambda, \underset{2}{\xi}{}^\lambda, \dots, \underset{n}{\xi}{}^\lambda$ and $\overset{1}{\varkappa}, \overset{2}{\varkappa}, \dots, \overset{n-1}{\varkappa}$ respectively, the Frenet equations of C may be written as

$$(1.2)\qquad \frac{\delta}{\delta s}\underset{a}{\xi}{}^\lambda = -\overset{a-1}{\varkappa}\underset{a-1}{\xi}{}^\lambda + \overset{a}{\varkappa}\underset{a+1}{\xi}{}^\lambda\,, \qquad (a = 1, 2, \dots, n;\ \overset{0}{\varkappa} = \overset{n}{\varkappa} = 0)\,,$$

where $\delta/\delta s$ denotes covariant differentiation with respect to arc length s along C.

A geodesic circle[1] is defined as a curve whose first curvature is constant and whose second curvature is identically zero. For such a geodesic circle, we have, from (1.2),

$$(1.3)\qquad \frac{\delta}{\delta s}\underset{1}{\xi}{}^\lambda = \overset{1}{\varkappa}\underset{2}{\xi}{}^\lambda\,,$$

$$(1.4)\qquad \frac{\delta}{\delta s}\underset{2}{\xi}{}^\lambda = -\overset{1}{\varkappa}\underset{1}{\xi}{}^\lambda\,,$$

where $\overset{1}{\varkappa}$ is a constant. Differentiating (1.3) covariantly and then substituting (1.4) in the obtained equation, we have

$$(1.5)\qquad \frac{\delta^2}{\delta s^2}\underset{1}{\xi}{}^\lambda = -(\overset{1}{\varkappa})^2\underset{1}{\xi}{}^\lambda\,.$$

The $\underset{1}{\xi}{}^\lambda$ denoting the unit tangent, we may put

$$\underset{1}{\xi}{}^\lambda = \frac{\delta u^\lambda}{\delta s}\,,$$

so that we have, from (1.3),

$$(\overset{1}{\varkappa})^2 = g_{\mu\nu}\frac{\delta^2 u^\mu}{\delta s^2}\frac{\delta^2 u^\nu}{\delta s^2}\,.$$

The equation (1.5) then becomes

$$(1.6)\qquad \frac{\delta^3 u^\lambda}{\delta s^3} + g_{\mu\nu}\frac{\delta^2 u^\mu}{\delta s^2}\frac{\delta^2 u^\nu}{\delta s^2}\frac{\delta u^\lambda}{\delta s} = 0\,.$$

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1) A. Fialkow: Conformal geodesics, Trans. Amer. Math. Soc. **45** (1939), 443–473.