29. *On Vector Lattice with a Unit.*

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§ 1. *Introduction and the theorems.* The purpose of the present note is to give a new proof to Kakutani-Krein's lattice-theoretic characterisation of the space of continuous functions on a bicompact space[1]. We first represent algebraically the vector lattice as point functions and then introduce the topology to the represented function lattice, while Kakutani-Krein's (independent and different) proofs both make use of the assumed norm and hence the conjugate space of the vector lattice. Our treatment may be compared with Birkhoff-Stone-Wallman's representation of Boolean algebra as field of sets or with Gelfand's[2] representation of normed ring as function ring[3].

A *vector lattice E* is a partially ordered real linear space, some of whose elements f are "non-negative" (written $f \geqq 0$) and in which

(V 1): If $f \geqq 0$ and $\alpha \geqq 0$, then $\alpha f \geqq 0$.
(V 2): If $f \geqq 0$ and $-f \geqq 0$, then $f=0$.
(V 3): If $f \geqq 0$ and $g \geqq 0$, then $f+g \geqq 0$.
(V 4): E is a lattice by the semi-order relation $f \geqq g$.

In this note we further assume the *Archimedean axiom*

(V 5): If $f > 0$ and $\alpha_n \downarrow 0$, then $\alpha_n f \downarrow 0$ (in order-topology),

and the existence of a *unit* $I > 0$ satisfying

(V 6): For any $f \in E$ there exists $\alpha > 0$ such that $-\alpha I \leqq f \leqq \alpha I$.

A linear subspace N of E is called an *ideal*[4] if N contains with f all x satisfying $|x| \leqq |f|$. Here we put, as usual, $|f| = f^+ - f^-$, $f^+ = f \vee 0 = \sup(f, 0)$, $f^- = f \wedge 0 = \inf(f, 0)$. An ideal $N \neq E$ is called *maximal* if it is contained in no other ideal $\neq E$. Denote by $\mathfrak{N}$ the set of all the maximal ideals N of E. It is proved below that the residual class E/N of E mod any maximal ideal N is linear-lattice-isomorphic to the vector lattice of real numbers, the non-negative elements $\in E$ and I respectively being represented by non-negative numbers and the number 1. We denote by $f(N)$ the real number which

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1) S. Kakutani: Proc. **16** (1940), 63-67. M. and S. Krein: C. R. URSS, **27** (1940), 427-430.

2) I. Gelfand: C. R. URSS, **23** (1939), 430-432.

3) After the present paper was completed, I knew, by Y. Kawada's remark, the paper of M. H. Stone: Proc. Nat. Acad. Sci. **27** (1941), 83-87, which arrived at our institute only recently. In this paper, Stone sketches a proof of Kakutani's theorem which is essentially the same as ours. (He seems to have not read Krein's paper.) Stone proves firstly the case of Banach vector lattice and then reduces the general case to it, while we first prove the algebraic case and then deduce from it the case of Banach vector lattice. It is to be noted that we do not, in the proof of the representation theorem, make use of the metrical nor order completeness other than the Archimedean axiom (V 5).

(4) Normal subspace in the terminology of Garrett Birkhoff: Lattice Theory, New York (1940).