PAPERS COMMUNICATED

28. *The Exceptional Values of Functions with the Set of Linear Measure Zero, of Essential Singularities.*

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1. Let $w=f(z)$ be a function which is meromorphic in a domain D except for a set E of essential singularities.

The object of this paper is to obtain some metrical property of the set of the values omitted by $f(z)$ near each point of E, provided that E is of Carathéodory's linear measure zero, the result of which is more precise than that given recently by M. L. Cartwright[1].

2. Let us begin by giving some preliminary definitions and notations. Given an increasing continuous function $h(t)$ defined for $t \geqq 0$ with $h(0)=0$, we shall denote, for each $\varepsilon > 0$, by $L_\varepsilon^h(A)$ the lower bound of all the sums $\sum_i h[\delta(A_i)]$ where $\{A_i\}_{i=1,2,\dots}$ is an arbitrary partition of a point-set A into a sequence of sets whose diameters $\delta(A_i)$ are all less than ε and no two of which have common points.

Making ε tend to zero, $L_\varepsilon^h(A)$ tends monotonically to a limit (finite or infinite) which is called the h-measure of A and denoted by $L^h(A)$. In particular, in the case that $h(t)=t^p$ $(p>0)$, we call this h-measure p-dimensional measure and write $L^{(p)}$ instead of L^{t^p}. One dimensional measure is also called (Carathéothory's) linear measure. Evidently, any set of Lebesgue's plane measure zero is equivalent to that of 2-dimentional measure zero.

As is well known, all these h-measures have the property of Carathéodory's outer measure[2].

3. In his very important and interesting paper entitled "On sufficient conditions for a function to be analytic," Besicovitch has shown among others that, *if E is of linear measure zero, then the function $f(z)$ is unbounded near each point of E.* He also proved from this that *the set of values taken by $f(z)$ near each point of E is everywhere dense*[3]. This is evidently a generalization of Weierstrass' classical theorem.

One might expect that this could be extended so far as to the theorem of Picard's type. Seidel has shown by an example that this is

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1) M. L. Cartwright. The exceptional values of functions with a non-enumerable set of essential singularities, Quart. J. Math., Vol. **8** (1937).

2) F. Hausdorff. Dimension und äusseres Mass, Math. Ann., Vol. **79** (1919).

3) A. S. Besicovitch. On sufficient conditions for a function to be analytic, and on behaviour of analytic functions in the neighbourhood of non-isolated singular points, Proc. London Math. Soc. (2), Vol. **32** (1931).

Though his result was first proved for regular functions, it may be needless to say that the same holds even for meromorphic functions.