70. On the Conformal Arc Length.

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(Comm. by T. TAKAGI, M.I.A., Oct. 11, 1941.)

Let us consider a curve C in a conformally connected manifold C_n whose conformal connection is defined by the formulae

$$\left\{\begin{array}{l} dA_0 = du^i A_i, \\ dA_j = \Pi^0_{jk} du^k A_0 + \Pi^i_{jk} du^k A_i + \Pi^\infty_{jk} du^k A_\infty, \\ dA_\infty = \Pi^i_{\infty k} du^k A_i, \end{array}\right. \tag{1}$$

where

$$\Pi^i_{\infty k} = g^{ij}\Pi^0_{jk}, \quad \Pi^\infty_{jk} = g_{jk} \quad \text{and} \quad g^{ij}g_{jk} = \delta^i_k. \tag{2}$$

$$(i, j, k, \dots = 1, 2, 3, \dots, n)$$

Defining two parameters s and t on the curve by the equations

$$g_{jk}\frac{du^j}{ds}\frac{du^k}{ds} = 1 \tag{3}$$

and

$$\{t, s\} = \frac{1}{2} g_{jk}\frac{\delta^2 u^j}{\delta s^2}\frac{\delta^2 u^k}{\delta s^2} - \Pi^0_{jk}\frac{du^j}{ds}\frac{du^k}{ds} \tag{4}$$

respectively, where

$$\{t, s\} = \frac{d^3t}{ds^3}\Big/\frac{dt}{ds} - \frac{3}{2}\left(\frac{d^2t}{ds^2}\Big/\frac{dt}{ds}\right)^2 \tag{5}$$

and

$$\frac{\delta^2 u^i}{\delta s^2} = \frac{d^2u^i}{ds^2} + \Pi^i_{jk}\frac{du^j}{ds}\frac{du^k}{ds}, \tag{6}$$

we find the following Frenet formulae[1)]

$$\left\{\begin{array}{l} \underset{(0)}{S} = \frac{dt}{ds}A_0, \quad \underset{(1)}{S} = \frac{d}{dt}\underset{(0)}{S}, \quad \underset{(2)}{S} = \frac{d}{dt}\underset{(1)}{S}, \\ \frac{d}{dt}\underset{(2)}{S} = -\overset{3}{\varkappa}\underset{(3)}{S}, \\ \frac{d}{dt}\underset{(3)}{S} = -\overset{3}{\varkappa}\underset{(0)}{S} + \overset{4}{\varkappa}\underset{(4)}{S}, \\ \frac{d}{dt}\underset{(4)}{S} = -\overset{4}{\varkappa}\underset{(3)}{S} + \overset{5}{\varkappa}\underset{(5)}{S}, \\ \dots\dots\dots\dots\dots\dots \\ \frac{d}{dt}\underset{(\infty)}{S} = -\overset{\infty}{\varkappa}\underset{(n)}{S}, \end{array}\right. \tag{7}$$

1) See, K. Yano, Sur la théorie des espaces à connexion conforme. Journal of the Faculty of Science, Tokyo Imperial University, Sec. I, Vol. IV, Part 1 (1939), 1–59.