12. *An Abstract Integral, VI.*

By Masahiro NAKAMURA.

Mathematical Institute, Tohoku Imperial University, Sendai.

(Comm. by M. FUJIWARA, M.I.A., Feb. 12, 1942.)

The purpose of this paper is to give an integral, similar to that of H. Freudenthal[1]. Our integral is defined for the functions with domains in a general lattice and range in a metric commutative semi-group.

1. We begin by the definitions and notations[2]:

[1.1] A is a metric commutative semi-group with zero elements whose operation is denoted by addition. And the addition is a contraction, i.e., $\delta(u+w, v+w) \leqq \delta(u, v)$, where $\delta(u, v)$ is the distance between u and v.

[1.2] L is a lattice with zero element.

[1.3] $f(x)$ is a one-valued function in L to A such as $f(0)=0$.

[1.4] A denumerable set $\{a_i\}=z(a)$ in L is called *resolution* of a if

1° $a_i > 0$, 2° $a_i \cap a_j = 0$ if $i \neq j$, 3° $\bigvee a_i = a$,

4° $\{a_i\}$ generates a Boolean algebra $L(z(a))$.

[1.5] $Z(a)$ is the class of all resolutions $z(a)$ of a.

[1.6] $z(a) \leqq z'(a)$ if and only if $L(z(a)) \leqq L(z'(a))$, the latter inequality being set implication.

[1.7] $y(a)$ is a finite subset of $z(a)$.

[1.8] $Y(z(a))$ is a class of all $y(a)$ such that $y(a) \leqq z(a)$.

[1.9] If $Z(a)$ consists of only one trivial resolution $z(a)=\{a\}$, then a is called *trivially soluble.*

[1.10] $y(a) \leqq y'(a)$ if and only if $y'(a)$ includes $y(a)$ as set.

[1.11] $f(y(a)) = \sum_{a_i \in y(a)} f(a_i)$.

Under above definitions we have clearly,

(1.12) $Z(a)$ is a partially ordered system.

(1.13) $Y(z(a))$ is a Moore-Smith set.

[1.14] If $f(y(a))$ converges to $u \in A$ in the sense of Moore-Smith, then we denote $u=f(z(a))$.

2. Here we define an integral as follows:

[2.1] If $f(z(a))$ converges to a unique $v \in A$ in $Z(a)$ in the sense of G. Birkhoff[3], we denote

---

1) H. Freudenthal, Proc. Ned. Akad. Wet. Amsterdam, **39** (1936).

2) [ ] indicates axiom and definition, ( ) theorem.

3) G. Birkhoff and L. Alaoglu, Ann. of Math., **41** (1940), 293–309.