PAPERS COMMUNICATED

102. *On the Regular Vector Lattice.*

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Introduction. L. V. Kantrovitch introduced the notion of regularity[1] in vector lattice and applied it to the space of measurable functions. In § 1 of this paper, we prove that the regularity axiom is decomposed into two simple propositions. In the succeeding articles we prove many theorems in Kantrovitch's paper under weaker assumption.

§ 1. Let $\mathfrak{L}$ be a complete vector lattice. Then the regularity axiom due to Kantrovitch reads as follows:

If $E_n \subset \mathfrak{L}$ for $n=1, 2, \dots$ and $\sup E_n$ tends to a limit y, then for each n there exists a finite subset E'_n of E_n such that $\lim\limits_{n\to\infty} E'_n = y$.

For regular vector lattice $\mathfrak{L}$, two theorems hold as Kantrovitch shows.

I. If $y_i^{(k)} \to y_i\,(o)$ (as $k \to \infty$) and $y_i \to y\,(o)$ (as $i \to \infty$) in $\mathfrak{L}$, then there exists an increasing sequence of indices $k_1, k_2, \dots$ such that $y_i^{(k_i)} \to y\,(o)$ $(i \to \infty)$[2]

II. For any set $E \subset \mathfrak{L}$, there exists an enumerable subset E' of E such that $\sup E' = \sup E$[3]

Conversely, we can prove the following theorem.

Theorem 1.1. **I** and **II** imply the regularity axiom.

Proof. By **II**, for each E_n there exists an enumerable set $E'_n = \{y_{n,k}\}$ $k=1, 2, \dots$, such that $\sup E_n = \sup (y_{n,k})$ $k=1, 2, \dots$ If we put $y_n^{(k)} = \sup (y_{n,1}, \dots, y_{n,k})$, then $y_n^{(k)} \uparrow \sup E_n$ $(n \to \infty)$. Therefore, if $\lim\limits_{n\to\infty} \sup E_n = y_0$, then by **I** we can find an increasing sequence of indices $\{k_n\}$ such that $\lim\limits_{n} y_n^{(k_n)} = y_0$. Hence $\lim \sup (y_{n,1}, \dots, y_{n,k_n}) = \lim\limits_{n\to\infty} \sup E_n$.

From the proof it is easy to see that in **II** we can replace the condition $y_i^{(k)} \to y_i\,(o)$ (as $k \to \infty$) by $y_n^{(k)} \uparrow y_n\,(o)$ $(k \to \infty)$.

In the space of measurable functions (S), (o)-convergence is equivalent to almost everywhere convergence[4]. Therefore, **I** is nothing but Fréchet's theorem[5].

We can easily verify that the space (S) satisfies **II**. But more generally we can prove

Theorem 1.2. **II** holds in the space of functions with metric function ρ such that 1°. for any $y \geqq 0$, $\rho(y)$ is defined and $\geqq 0$ and $\rho(y) = 0$

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1) L. V. Kantrovitch: Lineare halbgeordnete Räume, Recueil Math., **44** (1937), pp. 121-165.

2) loc. cit., Satz 24.

3) loc. cit., Satz 23, a).

4) G. Birkhoff, Lattice theory, Chapter VII.

5) M. Fréchet, Rendiconti di Palermo, **22** (1906), p. 15.