2. *On the Axioms of the Theory of Lattice.*

By Masatada KOBAYASI.

Mathematical Seminary, Hokkaido Imperial University, Sapporo.

(Comm. by T. TAKAGI, M.I.A., Jan. 12, 1943.)

1. System $\sum$.

Suppose that there is a set S of elements, between each two of which two dualistic operations $\cup$ and $\cap$ are so defined that their results are unique and belong to S. If the two operations satisfy the following postulates:

L 1.	Idempotent law:	(a)	$x \cup x = x$	for all x,
		(b)	$x \cap x = x$	for all x,
L 2.	Commutative law:	(a)	$x \cup y = y \cup x$	for all x and y,
		(b)	$x \cap y = y \cap x$	for all x and y,
L 3.	Associative law:	(a)	$x \cup (y \cup z) = (x \cup y) \cup z$	for all x, y and z,
		(b)	$x \cap (y \cap z) = (x \cap y) \cap z$	for all x, y and z,
L 4.	Absorptive law:	(a)	$x \cup (x \cap y) = x$	for all x and y,
		(b)	$x \cap (x \cup y) = x$	for all x and y,

then the set S is called a lattice for the operations $\cup$ and $\cap$[1].

G. Köthe and H. Hermes showed that if L 4 is satisfied then L 1 does so (Enzyklopädie Bd. I-1 Heft 5, 13 (1939), and they took L 2–L 4 as the axioms for the lattice.

Now, we replace the Absorptive law L 4 by a weaker postulate, viz.[2],

L 4* (a) if $y \cup x = x$ then $y \cap x = y$,

(b) if $y \cap x = x$ then $y \cup x = y$;

and this, together with L 1 (a), L 2, L 3 will be taken as postulates for a "System $\sum$"[3].

We shall demonstrate the independency of postulates of $\sum$. Before doing so, we enumerate some relations between these postulates.

(I) L 1 (a) *and* L 4 (a) *imply* L 1 (b). In fact, by L 1 (a),

---

1) Cf. Ore, *On the Foundation of Abstract algebra 1*, Ann. Math., **36**, 409 (1935), Philip M. Whitmann, *Free lattices*, Ann. Math., **42**, 325 (1941), and G. Birkhoff, *Lattice theory*, Ammer. Math. Soc. Coll. Pub. XXV (1940), etc.

2) In the case when L 2 holds, we may use instead of L 4* (a) any one of the following three postulates: (1) *if* $x \cup y = x$ *then* $y \cap x = y$, (2) *if* $y \cap x = x$ *then* $x \cap y = y$, (3) *if* $x \cup y = x$ *then* $x \cap y = y$, and also instead of L 4 (b) any one of the three postulates: (1) if $x \cap y = x$ then $y \cup x = y$, (2) if $y \cap x = x$ then $x \cup y = y$, (3) if $x \cap y = x$ then $x \cup y = y$.

3) In this "System $\sum$", we may use the postulate L 1 (b), instead of L 1 (a). Cf. (1).

Then by Footnote 2) there are 32 equivalent systems of postulates for a lattice.