38. *On the Duality Theorem of Non-commutative Compact Groups.*

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1. The duality theorem of L. Pontrjagin[1] concerning the compact commutative group was, by T. Tannaka[2], ingeneously extended to arbitrary compact group. To this extension, S. Bochner[3] and M. Krein[4] respectively gave new proofs recently. They three all start with the proof of the positivity or the continuity of certain homomorphisms. The proof given below is a direct one and will be shorter than theirs. It may be considered as a simplification of Tannaka's original proof. Our method lies in the use of Gelfand-Silov's abstraction[5] of Weierstrass' polynomial approximation theorem.

2. Let $\mathfrak{G}$ be a compact (=bicompact) Hausdorff group and let $\mathfrak{U}$ be a complete set of mutually inequivalent, continuous, unitary, irreducible representations $U(s)=\big(u_{ij}(s)\big)$ of $\mathfrak{G}$. The completeness (Peter-Weyl-Neumann's theory of almost periodic functions) implies: 1) for any pair of distinct points $s, t\in\mathfrak{G}$, there exists an $U(s)\in\mathfrak{U}$ such that $U(s)\neq U(t)$, 2) if $U_1(s)$, $U_2(s)\in\mathfrak{U}$ then the *product* (*complex conjugate*) *representation* $U_1(s)\times U_2(s)\big(\bar{U}_1(s)\big)$ is, as a unitary representation of $\mathfrak{G}$, completely reducible to a sum of a finite number of representations $\in\mathfrak{U}$. Let $\mathfrak{R}$ be the totality of *Fourier polynomials*:

$$x(s)=\sum a_{ij}^{(\gamma)}u_{ij}^{(\gamma)}(s)\,,$$

viz. finite linear combinations of $u_{ij}^{(\gamma)}(s)$, where $\big(u_{ij}^{(\gamma)}(s)\big)\in\mathfrak{U}$ and $a_{ij}^{(\gamma)}$ denote complex numbers. By 2) $\mathfrak{R}$ is a ring with unit e $\big(e(s)\equiv1$ on $\mathfrak{G}\big)$ and complex multipliers. Here the sum and the multiplication in $\mathfrak{R}$ is the ordinary function sum and function multiplication. Let $\mathfrak{T}$ be the totality of the linear homomorphisms T of $\mathfrak{R}$ onto the field $\mathfrak{K}$ of complex numbers such that

$$(1)\quad\begin{cases} T\cdot e=1\,, \\ T\cdot\bar{x}=\overline{T\cdot x} \quad \big(\text{bar indicates complex conjugates: } \bar{x}(s)=\overline{x(s)}\big). \end{cases}$$

$\mathfrak{T}$ is not void since each $s\in\mathfrak{G}$ induces such a homomorphism T_s:

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1) Topological groups, Princeton (1939).

2) Über den Dualitätssatz der nichtkommutativen topologischen Gruppen, Tôhoku Math. J., **45** (1938).

3) 位相數學, 第 **4** 卷, 第 **1** 號 (昭和 17 年).

4) On positive functionals on almost periodic functions, C. R. URSS, **30** (1941).

5) Über verschiedene Methoden der Einführung der Topologie in die Menge der maximalen Ideale eines normierten Ringes, Rec. Math., **9, 7** (1941). Cf. H. Nakano: 連續函數 ring 及ビ vector lattice, 全國紙上數學談話會 **218** (1941).