70. On the Theory of Hypersurfaces in the Path-space of the Third Order.

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§ 0. The theory of path-space of the third order has been developed by Prof. H. Hombu[1]. In the present note, we shall deal with the theory of hypersurfaces in such a space. In an n-dimensional manifold V_n referred to a coordinate system x^λ $(\lambda=1, 2, \dots, n)$, let us consider a system of paths, defined by the differential equations of the third order,

$$T^\lambda = x^{(3)\lambda} + H^\lambda(x, x^{(1)}, x^{(2)}) = 0\,. \tag{0.1}$$

In order that our system of paths admits of projective parameters, it is necessary that

$$\text{(a)}\quad H^\lambda_{(1)\nu}x^{(1)\nu} + 2H^\lambda_{(2)\nu}x^{(2)\nu} = 3H^\lambda\,, \qquad \text{(b)}\quad H^\lambda_{(2)\nu}x^{(1)\nu} = -3x^{(2)\lambda}\,. \tag{0.2}$$

The base connections of our V_n are defined by

$$\text{(a)}\quad \delta x^{(1)\lambda} = dx^{(1)\lambda} + \frac{1}{3}H^\lambda_{(2)\nu}dx^\nu\,, \tag{0.3}$$

$$\text{(b)}\quad \delta x^{(2)\lambda} = dx^{(2)\lambda} + \frac{2}{3}H^\lambda_{(2)\nu}dx^{(1)\nu} + \frac{1}{3}H^\lambda_{(1)\nu}dx^\nu\,.$$

We see that $\dfrac{\delta x^{(1)\lambda}}{dt}=0$ (along any curve) and $\dfrac{\delta x^{(2)\lambda}}{dt}=0$ (along paths).

The covariant derivative of a vector v^λ in V_n is given by

$$\delta v^\lambda = dv^\lambda + w^\lambda_\mu v^\mu\,, \tag{0.4}$$

where

$$w^\lambda_\mu = \underset{(0)}{\overset{*}{\Gamma}}{}^\lambda_{\mu\nu}dx^\nu + \underset{(1)}{\overset{*}{\Gamma}}{}^\lambda_{\mu\nu}\delta x^{(1)\nu}\,,$$

$$\text{(a)}\quad \underset{(0)}{\overset{*}{\Gamma}}{}^\lambda_{\mu\nu} = \frac{1}{3}H^\lambda_{(2)\mu(1)\nu} - \frac{2}{9}H^\lambda_{(2)\mu(2)\sigma}H^\sigma_{(2)\nu}\,, \qquad \text{(b)}\quad \underset{(1)}{\overset{*}{\Gamma}}{}^\lambda_{\mu\nu} = \frac{2}{3}H^\lambda_{(2)\mu(2)\nu}\,. \tag{0.5}$$

The equation (0.4) can be also written as follows:

$$\delta v^\lambda = \nabla^{(0)}_\nu v^\lambda \cdot dx^\nu + \nabla^{(1)}_\nu v^\lambda \cdot \delta x^{(1)\nu} + \nabla^{(2)}_\nu v^\lambda \cdot \delta x^{(2)\nu}\,,$$

where

$$\nabla^{(0)}_\nu v^\lambda = \bar{\nabla}^{(0)}_\nu v^\lambda + \underset{(0)}{\overset{*}{\Gamma}}{}^\lambda_{\mu\nu}v^\mu\,, \quad \nabla^{(1)}_\nu v^\lambda = \bar{\nabla}^{(1)}_\nu v^\lambda + \underset{(1)}{\overset{*}{\Gamma}}{}^\lambda_{\mu\nu}v^\mu\,, \quad \nabla^{(2)}_\nu v^\lambda = \bar{\nabla}^{(2)}_\nu v^\lambda\,, \tag{0.6}$$

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1) H. Hombu: Projektive Transformation eines Systems der gewöhnlichen Differentialgleichungen dritter Ordnung. Proc. **13** (1937), 187–190, Die projektive Theorie der "paths" 3-ter Ordnung. Proc. **14** (1938), 36–40.