# 87. On the Representation of Boolean Algebra.

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**1.** Representation theory of Boolean algebra was developed by Stone, Wallman and many writers. Wallman's[1] method is simpler than that of Stone[2] in the point that the notion of ideal is not used. The method of Livenson[3] is complicated than that of Wallman. But if we replace the regular table of Livenson by the set satisfying conditions (1°), (2°) and (4°) in § 2, then the maximal regular table becomes a ideal basis. Further we can prove that the representation space of Livenson becomes a $T_2$-space satisfying the first countability axiom.

**2.** Let $L$ be a distributive lattice including 0 and 1. That is, $L$ is a lattice having zero element 0 and unit element 1 and for any three elements $a$, $b$ and $c$

$$a(b \vee c) = ab \vee ac \quad \text{and} \quad a \vee bc = (a \vee b)(a \vee c)\,.$$

Now we consider a subset $\{g\}$ of $L$ satisfying the following conditions:

(1°) $0 \bar{\in} \{g\}$.

(2°) If $g_1, g_2 \in \{g\}$ then there exists $g_3$ such that $g_3 < g_1 g_2$.

In such two sets $\{g\}$ and $\{g'\}$, if for any $g \in \{g\}$ there exists $g' \in \{g'\}$ such that $g' < g$ then we write

$$\{g\} \prec \{g'\}\,.$$

Further we will introduce two conditions concerning $\{g\}$ in $L$:

(3°) For $\{g\}$ and any two elements $a$ and $b$ such as $g(a \vee b) = g$ there exists $g_1 \in \{g\}$ such that $g_1 a = g_1$ or $g_1 b = g_1$.

(4°) For $\{g\}$ and any $a \in L$ there exists $g \in \{g\}$ satisfying $ag = g$ or $ag = 0$.

*Lemma 1.* Under (1°) and (2°), (4°) implies (3°).

Suppose that $\{g\}$ satisfies (1°), (2°) and (4°) and $a$ and $b$ are any elements satisfying $(a \vee b)g = g$ for some $g \in \{g\}$. Then there exist $g_1$ and $g_2$ such that $ag_1 = g_1$ or $ag_1 = 0$ and $bg_2 = g_2$ or $bg_2 = 0$. If $ag_1 = bg_2 = 0$, then $g_3 < g < a \vee b$ for $g_3 < gg_1g_2$. Hence $0 = ag_3 \vee bg_3 = (a \vee b)g_3 = g_3$. This is a contradiction.

*Lemma 2.* Suppose that $\{g\}$ satisfies (3°) (or (4°)) and $\{g\} \prec \{g'\} \prec \{g\}$. Then $\{g'\}$ satisfies (3°) (or (4°)).

Suppose that $\{g\}$ satisfies (3°) and that $a$ and $b$ are any two elements satisfying $(a \vee b)g' = g'$ for some $g' \in \{g'\}$. If $gg' = g$ then $g(a \vee b) = g$. Consequently there exists $g_1 \in \{g\}$ such that $ag_1 = g_1$ or

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1) H. Wallman, Lattice and topological Spaces (Ann. Math., Vol. 39 (1938)).

2) H. Stone, Topological Representations of Distributive Lattice and Browerian Logics. (Casopic pro pestovani matematiky a fysiky 1939).

3) E. Livenson, On the realization of Boolean algebras by algebras of sets (Rec. Math. de la Soc. Math. de Moscou (1940)).