58. *On Quasi-Evaluations of Compact Rings.*

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(Comm. by T. TAKAGI, M.I.A., May 12, 1944.)

Recently H. Anzai has proved that every compact associative ring containing no left-(or right-)total nul-divisor is totally disconnected and is a limit ring of finite rings[1]. In the present paper the author proposes to prove that every compact ring containing no total nul-divisor has "sufficiently many quasi-evaluations." These evaluations serve to clarify the uniform structure of compact rings, and the author has in view to apply this result, in a forthcoming paper[2], to determine the structure of locally compact totally disconnected fields.

To prepare the introduction of the quasi-evaluations in § 2, we shall, in § 1, consider the compact open ideals of the ring determined by finite sets of characters of the additive group of the ring, by means of which Anzai's result is easily deduced in a more precise form.

§ 1. Let $\mathfrak{R}$ be a compact associative topological ring, that is, an associative ring forming a compact topological group with respect to addition, where the product xy is continuous in two variables x and y.

Denote by $\mathfrak{R}^*$ the character group (in the sense of Pontrjagin-van Kampen[3]) of the compact additive group $\mathfrak{R}$, then $\mathfrak{R}^*$ is a discrete group, and

(1) $V(0\,;\xi_1,\xi_2,\dots,\xi_r\,;1/m)=\{x\,|\,|(x,\xi_i)|<1/m,\ i=1,2,\dots,r\}$,

where $\xi_1,\xi_2,\dots,\xi_r\in\mathfrak{R}^*$, $m=1,2,\dots$, $r=1,2,\dots$, form a complete system of neighbourhoods of zero in $\mathfrak{R}$[4]. Define further

$$[a]'=a\mathfrak{R}+\mathfrak{R}a+\mathfrak{R}a\mathfrak{R}$$

for $a\in\mathfrak{R}$, and denote by $[a]$ the additive subgroup $a\mathfrak{R}+\mathfrak{R}a+\Sigma\mathfrak{R}a\mathfrak{R}$ generated by $[a]'$, similarly for $A\leq\mathfrak{R}$ by $[A]$ the subgroup generated by all $[a]'$ with $a\in A$. Then we have

(2) $x\in[a]'$ implies $tx=x+x+\cdots+x$ (t times) $\in[a]'$

for any integer $t(>0)$, since $tx=t(ax_1+x_2a+x_3ax_4)=a(tx_1)+(tx_2)a+(tx_3)ax_4\in[a]'$; and

1) H. Anzai, On compact topological rings, Proc. **19** (1943), 616.

2) Y. Otobe, On locally compact fields.

3) E. R. van Kampen, Locally bicompact abelian groups and their character groups, Ann. of Math., **36** (1935).

4) We denote by (x,ξ) the value (in real numbers mod 1) of the continuous additive character ξ at $x\in\mathfrak{R}$. For subsets $A,B\leq\mathfrak{R}$ and $C^*\leq\mathfrak{R}^*$, $(A,C^*)=0$ means that $(x,\xi)=0$ for all $x\in A$, $\xi\in C^*$. We put $A+B=\{x+y\,|\,x\in A,\ y\in B\}$, $AB=\{xy\,|\,x\in A,\ y\in B\}$, $\Sigma A=\{x_1+x_2+\cdots+x_r\,|\,x_i\in A\ (i=1,\dots,r),\ r=1,2,\dots\}$, etc. The annihilator $\{\mathfrak{R},C^*\}=\{x\,|\,(x,C^*)=0\}$ or $\{\mathfrak{R}^*,A\}=\{\xi\,|\,(A,\xi)=0\}$ is a closed additive subgroup of $\mathfrak{R}$ or $\mathfrak{R}^*$ respectively.