71. *The Distribution of Grouped Moments in Large Samples.*

By Tatsuo KAWATA.

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1. We divide the whole interval $(-\infty, \infty)$ into subintervals of length δ, which we denote I_a, $a = \dots -1, 0, 1, 2, \dots$. Let I_0 contain the origin and the distance of the origin and the center of I_0 be t. Thus we can write $I_a = \left(\left(a-\frac{1}{2}\right)\delta + t, \left(a+\frac{1}{2}\right)\delta + t\right)$. Now consider a sample of size n from a certain population and let the number of individuals of the sample which fall into I_a be n_a. For this grouping, consider the sample moment of the r-th order.

$$(1) \qquad {}_\delta M_r = \sum_{a=-\infty}^{\infty} \frac{n_a}{n}(a\delta + t)^r .$$

We assume that the population variable has the finite $2r$-th moment and let its probability density be $f(x)$. Then the probability that an individual falls into I_a is

$$p_a = \int_{I_a} f(x)dx .$$

The mean value of the random variable ${}_\delta M_r$ is

$$(2) \qquad {}_\delta \mu'_r = E({}_\delta M_r) = \sum_{a=-\infty}^{\infty} p_a(a\delta + t)^r \text{ [1]} .$$

Then under suitable conditions, we have

$$(3) \qquad {}_\delta\mu'_1 \fallingdotseq \mu'_1 , \quad {}_\delta\mu'_2 \fallingdotseq \mu'_2 + \frac{\delta^2}{12} , \quad {}_\delta\mu'_3 \fallingdotseq \mu_3 + \delta^2 \cdot \frac{\mu'_1}{4} ,$$

$${}_\delta\mu'_4 \fallingdotseq \mu'_4 + \delta \cdot \frac{\mu'_2}{2} + \frac{\delta^4}{80} , \dots$$

where μ'_r is the r-th moment of the population variable[2]. The relation (3) is known as Sheppard's correction.

The object of this paper is to discuss the sampling error of ${}_\delta M_r$ in the large sample or in other words, the limit distribution of the variable ${}_\delta M_r$ as $n \to \infty$.

2. Let X $(\dots, X_{-1}, X_0, X_1, X_2, \dots)$ be a point in a space of infinite dimensions Ω and X_a take either 0 or 1. Let the probability that X_a takes 1 be p_a. In the space we define the probability such that the probability that X takes a point of the enumerable set $\{x^{(a)}\}$ $(a = \dots, -1, 0, 1, 2, \dots)$ is p_a and the probability that X is a point of a set which does not contain a point of $\{x^{(a)}\}$ is 0, where

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1) For the meaning of the mean value, we shall clarify it in the following lines

2) S. S. Wilks, Statistical inferences. Princeton Lecture, 1937.