95. *Equivalence of Two Topologies of Abelian Groups.*

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Let G be a locally compact (=bicompact), separable abelian group and let X be the totality of continuous characters[1] $\chi(g)$ of G. It is well known[2] that X is also a locally compact, separable abelian group by the multiplication

$$\chi_1\chi_2(g)=\chi_1(g)\chi_2(g)$$

and by Pontrjagin's topology induced from the (closed) neighbourhood:

$$U(\chi_1)=\{\chi;\ \sup_{g\in G_0}|\chi(g)-\chi_1(g)|\leqq\varepsilon,\qquad G_0=\text{compact subset of } G\}.$$

X also constitutes a locally compact, separable topological space $\widetilde{X}$ by the topology induced from the (closed) neighbourhood:

$$\widetilde{V}(\chi_1)=\left\{\chi;\left|\int_G x_i(g)\chi(g)dg-\int_G x_i(g)\chi_1(g)dg\right|\leqq\varepsilon,\quad i=1,2,\ldots,n\right\}$$

where $x_i(g)\in L_1(G)$ viz. $x_i(g)$ denote measurable functions integrable over G with respect to Haar's invariant measure dg on G. The latter topology is introduced by I. Gelfand and D. Raikov[3], and its equivalence to Pontrjagin's topology plays a fundamental rôle in the ring-theoretic treatment and extension of the classical Fourier analysis based upon the theory of normed ring[4]. However the proof of the equivalence is, so far as we know, not published by the Russian school, though stated and used by them repeatedly[5].

The purpose of the present note is i): to give it a proof and ii) to show that the character group is a topological group in Gelfand-Raikov's topology even when G is not separable. For the purpose we make use of the following

Lemma. For any χ_2, the mapping

$$\chi\longrightarrow\chi_2\chi$$

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1) A continuous character of G is a continuous homomorphic mapping of G in the topological group of rotations of a circle.

2) L. Pontrjagin: Topological group, Princeton (1939), 127.

3) C. R. URSS, **28**, 3 (1940).

4) D. Raikov: C. R. URSS, **28**, 4 (1940). M. Krein: C. R. URSS, **30**, 6 (1941). D. Raikov: C. R. URSS, **30**, 7 (1941). K. Yosida: Proc. **20** (1944), 269. The author (Yosida) wishes to withdraw the §3 of this note, since the Lemma 2 is valid for $z\in L_1(G)$ only and thus the arguments in §3 is insufficient. A complete proof and the fact that Bochner-Raikov's theorem may be derived from Plancherel's theorem will be published elsewhere.—During the proof, Y. Kawada kindly communicated that 3° may be obtained from Bochner-Raikov's theorem.

5) H. Anzai kindly communicated M. Fukamiya's unpublished proof of the equivalence, which is entirely different from ours.