122. *Free Topological Groups and Infinite Direct Product Topological Groups.*

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1. The notion of a free topological group was introduced by A. Markoff[1], and the existence of a free topological group for any completely regular topological space was established by him. Recently, further investigations were given to this problem by T. Nakayama[2], who among other things proved that every free topological group is maximally almost periodic. The purpose of this note is to give a simple and direct proof to the existence theorem of A. Markoff. Our method of proof is based on the use of an infinite direct product topological group, and will make it possible to obtain many other known and new results concerning free topological groups almost immediately.

2. A topological space R is a *subspace* of another topological space S, if R is a subset of S and if the relative topology induced on R by that of S is equivalent with the original topology of R. It was proved by E. Čech[3] that, *for any completely regular topological space R, there exists a compact Hausdorff space $\bar{R}$ with the following properties:*

(1) *R is a subspace of $\bar{R}$,*

(2) *R is dense in $\bar{R}$,*

(3) *for any continuous mapping $\varphi(x)$ of R into any compact Hausdorff space S, there exists a continuous mapping $\Phi(x)$ of $\bar{R}$ into S such that $\Phi(x)=\varphi(x)$ on R.*

Such a compact Hausdorff space $\bar{R}$ is uniquely determined up to a homeomorphism which leaves every element of R invariant. $\bar{R}$ is called the Čech compactification of R.

A topolological group T is *toroidal* if it is topologically isomorphic with the infinite direct product topological group $\boldsymbol{P}_{\gamma\in\Gamma}K_\gamma$, where each K_γ is a topological group topologically isomorphic with the additive group K of all real numbers mod 1 with the usual compact topology.

Lemma 1. Every completely regular topological space is a subspace of a toroidal group.

Proof. Let R be a completely regular topological space. Let $\{\varphi_\gamma\,|\,\gamma\in\Gamma\}$ be the family of all continuous mappings $\varphi_\gamma(x)$ of R into K. Consider the toroidal group $T=\boldsymbol{P}_{\gamma\in\Gamma}K_\gamma$, where $K_\gamma=K$ for each

1) A. Markoff, On free topological groups, C. R. URSS, **31** (1941), 299–301.

2) T. Nakayama, Note on free topological groups, Proc., **19** (1943), 471–475.

3) E. Čech, On bicompact spaces, Annals of Math., **38** (1937).
S. Kakutani, Concrete representation of abstract (M)-spaces and the characterization of the space of continuous functions, Annals of Math., **42** (1941).