142. *Subprojective Transformations, Subprojective Spaces and Subprojective Collineations.*

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§ 1. *The subpaths.*

Let A_n be an affinely connected space of n dimensions whose components of connection are $\Pi^\lambda_{\mu\nu}(x)$.

If we consider a curve $x^\lambda = x^\lambda(r)$ in this space, the derivative of $x^\lambda(r)$ with respect to the parameter r

$$\frac{\delta x^\lambda}{\delta r} = \frac{dx^\lambda}{dr}$$

defines the direction of the tangent at a point x^λ of the curve, but the covariant derivative

$$\frac{\delta^2 x^\lambda}{\delta r^2} = \frac{d^2 x^\lambda}{dr^2} + \Pi^\lambda_{\mu\nu}\frac{dx^\mu}{dr}\frac{dx^\nu}{dr}$$

of the tangent vector $\frac{dx^\lambda}{dr}$ does not define a direction uniquely. For, if we change the parameter r into $\bar{r}$, the vector $\frac{\delta^2 x^\lambda}{\delta \bar{r}^2}$ becomes a linear combination of $\frac{\delta^2 x^\lambda}{\delta r^2}$ and $\frac{\delta x^\lambda}{\delta r}$. Thus two vectors $\frac{\delta^2 x^\lambda}{\delta r^2}$ and $\frac{\delta x^\lambda}{\delta r}$ define, independently of the choice of the parameter r, a two dimensional linear space. We shall call it osculating plane defined along the curve. If the curve is a so-called path the osculating plane is indeterminate.

Now, we suppose that there is given a contravariant vector field $\xi^\lambda(x)$ in our affinely connected space A_n and shall consider a system of curves whose osculating planes contain always the contravariant vector field ξ^λ. The differential equations of such curves are

$$\frac{d^2 x^\lambda}{dr^2} + \Pi^\lambda_{\mu\nu}\frac{dx^\mu}{dr}\frac{dx^\nu}{dr} = \alpha\frac{dx^\lambda}{dr} + \beta\xi^\lambda. \tag{1.1}$$ [1]

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1) The equations of this type have first appeared in D. van Dantzig's projective geometry. See, for example, D. van Dantzig: Theorie des projektiven Zusammenhangs n-dimensionaler Räume. Math. Ann. **106** (1932), 400-454. J. A. Schouten and J. Haantjes: Zur allgemeinen projektiven Differentialgeometrie, Compositio Math. **3** (1936), 1-51. J. Haantjes: On the projective geometry of paths, Proc. of the Edinburgh Math. Soc. **5** (1937), 103-115. The paths in these theories are represented by subpaths in an affinely connected space A_{n+1} of $n+1$ dimensions which represents the projectively connected space P_n. The present author showed that the paths in O. Veblen's projective space may also be represented by subpaths in an affinely connected space A_{n+1} of $n+1$ dimensions which represents the projective space of n dimensions. See, K. Yano: Sur les équations des paths dans l'espace projcctif généralisé de M. O. Veblen. To appear in the Proc. Physico-Math. Soc. Japan, **26** (1944).