Periodic points on T-fiber bundles over the circle

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Introduction

Let $f : M \to M$ be a map and $x \in M$, where M a compact manifold. The point x is called a periodic point of f if there exists $n \in \mathbb{N}$ such that $f^n(x) = x$, in this case x a periodic point of f of period n. The set of all $\{x \in M |$ x is periodic$\}$ is called the set of periodic points of f and is denoted by $P(f)$.

If M is a compact manifold then the Nielsen theory can be generalized to periodic points. Boju Jiang introduced (Chapter 3 in [1]) a Nielsen-type homotopy invariant $NF_n(f)$ being a lower bound of the number of n-periodic points, for each g homotopic to f; $Fix(g^n) \geq NF_n(f)$. In case $dim(M) \geq 3$, M compact PL- manifold, then any map $f : M \to M$ is homotopic to a map g satisfying $Fix(g^n) = NF_n(f)$, this was proved in [2].

Consider a fiber bundle $F \to M \xrightarrow{p} B$ where F, M, B are closed manifolds and $f : M \to M$ a fiber-preserving map over B. In natural way is to study periodic points of f on M, that is, given $n \in \mathbb{N}$ we want to study the set $\{x \in M \mid f^n(x) = x\}$. The our main question is; when f can be deformed by a fiberwise homotopy to a map $g : M \to M$ such that $Fix(g^n) = \emptyset$?

This paper is organized into three sections besides one. In Section 1 we describe our problem in the general context of fiber bundle with base and fiber closed manifolds.

In section 2, given a positive integer n and a fiber-preserving map $f : M \to M$, in a fiber bundle with base circle and fiber torus, we present necessary and sufficient conditions to deform $f^n : M \to M$ to a fixed point free map over S^1, see Theorem 2.3. In the Theorem 2.4 we described linear models of maps,

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