A controversy and the writing of a history The discussion of "small oscillations" (1760-1860) from the standpoint of the controversy between Jordan and Kronecker (1874)

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Camille Jordan and Leopold Kronecker were having a great controversy throughout the whole year of 1874, a controversy originally caused by Jordan's ambition to reorganise the theory of bilinear forms through what he designated as the "simple" notion of "canonical form":

> It is known that any bilinear polynomial $P = \sum A_{\alpha\beta} x_\alpha y_\beta$ ($\alpha = 1, 2, \ldots, n$; $\beta = 1, 2, \ldots, n$) can be reduced to its canonical form $x_1 y_1 + \ldots + x_m y_m$, by linear transformations applied to the two sets of variables $x_1, \ldots, x_n, y_1, \ldots, y_n$. We now consider the following questions:
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> 1. To reduce a bilinear polynomial P to its canonical form by orthogonal substitutions applied to the two sets of variables $x_1, \ldots, x_n; y_1, \ldots, y_n$.
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> 2. To reduce a bilinear polynomial P to its canonical form by the use of the same substitution on the x's and the y's.
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> 3. To reduce simultaneously two bilinear polynomials P and Q to a canonical form.
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> [...] The third problem has already been solved by M. Weierstrass [...] the solutions given by the geometers from Berlin are, nevertheless, incomplete; we will therefore suggest an extremely simple new method that holds no exception [...]. We will show that the problem of the simultaneous reduction of two functions P and Q is identical to the problem of

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