The sets closest to ovoids in $Q^-(2n+1,q)$

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1 Introduction

An *ovoid* of a polar space is a set of points with the property that every maximal subspace contains exactly one point of it. The existence of ovoids in polar spaces was studied extensively, see for example [5, 2, 3, 4] and the overview in [1, Appendix VI]. Clearly, if a polar space contains an ovoid $\mathcal{O}$, then $|\mathcal{O}|$ is the minimum size of a set of points that meets every maximal subspace of that polar space.

By $Q^-(2n+1,q)$ we denote the elliptic quadric of $\mathrm{PG}(2n+1,q)$. An ovoid of $Q^-(2n+1,q)$ has $q^{n+1}+1$ points [1]. However, Thas [5] has shown that $Q^-(2n+1,q)$ has no ovoids for $n \geq 2$. We will improve this result by showing that $q^{n+1}+q^{n-1}$ is the minimum cardinality of a set of points that meets every maximal subspace of $Q^-(2n+1,q)$. More precisely, we prove the following theorem.

Theorem 1.1 *Let B be a set of points of $Q = Q^-(2n+1,q)$ such that every maximal subspace of Q has a point in B. Let $\perp$ be the related polarity of $\mathrm{PG}(2n+1,q)$. Then $|B| \geq q^{n+1}+q^{n-1}$ with equality if and only if $B = (U^\perp \setminus U) \cap Q$ for a subspace U of dimension $n-2$ with $U \subseteq Q$.*

If U is a subspace of Q of dimension $n-2$, then the set $B := (U^\perp \setminus U) \cap Q$ meets all maximal subspaces of Q. For, if S is a maximal subspace of Q, then $\dim(S \cap U^\perp) = 1 + \dim(S \cap U)$ and thus $S \cap U \neq \emptyset$.

Notice that the quotient space $U^\perp/U$ is a 3-space and that Q induces a $Q^-(3,q)$ on this 3-space (that is the set $\{\langle U,P\rangle \mid P \in (U^\perp \setminus U) \cap Q\}$ is a $Q^-(3,q)$ of $U^\perp/U$). Thus $B := (U^\perp \setminus U) \cap Q$ has cardinality $(q^2+1)q^{n-1}$.

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