Convergence of difference analogues to the Darboux problem with functional dependence

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Abstract

We consider the Darboux problem with functional dependence for z, $D_x z$ and $D_y z$ on the right-hand side of the differential equation. We investigate a wide class of difference schemes for the differential-functional problem. In the present paper we prove convergence theorems by means of consistency and stability statements.

1 Introduction

Take a, $b > 0$ and α, $\beta \geq 0$. Define $E = [0, a] \times [0, b]$, $E^0 = [-\alpha, a] \times [-\beta, b] \backslash (0, a] \times (0, b]$, and $B = [-\alpha, 0] \times [-\beta, 0]$ Given a function $z : E^0 \cup E \to R$ and a point $(x, y) \in E$, we define the functional $z_{(x,y)} : B \to R$ by $z_{(x,y)}(\xi, \eta) = z(x+\xi, y+\eta)$ for $(\xi, \eta) \in B$. Suppose that we are given a function $f : \Omega := E \times X_0 \times X_1 \times X_2 \to R$, where X_0, X_1, X_2 are some subsets of the set of all functions from B to R. Take a differentiable function $\phi : E^0 \to R$. Consider the Darboux problem

$$D_{xy} z(x, y) = f\Big(x, y, z_{(x,y)}, (D_x z)_{(x,y)}, (D_y z)_{(x,y)}\Big), \tag{1}$$

$$z(x, y) = \phi(x, y), \qquad (x, y) \in E^0. \tag{2}$$

We will assume that there exists a classical solution to problem (1), (2), i.e. a continuous function $v : E^0 \cup E \to R$ which satisfies (2) on E^0, is of class C^2 on E, and satisfies (1) on the set E.

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