Unordered Baire-like vector-valued function spaces.

J.C. Ferrando*

Abstract

In this paper we show that if I is an index set and X_i a normed space for each $i \in I$, then the ℓ_p-direct sum $(\oplus_{i\in I} X_i)_p, 1 \leq p \leq \infty$, is UBL (unordered Baire-like) if and only if $X_i, i \in I$, is UBL. If X is a normed UBL space and (Ω, Σ, μ) is a finite measure space we also investigate the UBL property of the Lebesgue-Bochner spaces $L_p(\mu, X)$, with $1 \leq p < \infty$.

In what follows (Ω, Σ, μ) will be a finite measure space and X a normed space. As usual, $L_p(\mu, X), 1 \leq p < \infty$, will denote the linear space over the field $\mathbb{K}$ of the real or complex numbers of all X-valued μ-measurable p-Bochner integrable (classes of) functions defined on Ω, provided with the norm

$$\|f\| = \left\{ \int_\Omega \|f(\omega)\|^p \, d\mu(\omega) \right\}^{1/p}$$

When $A \in \Sigma, \chi_A$ will denote the indicator function of the set A.

On the other hand, if $\{X_i, i \in I\}$ is a family of normed spaces, we denote by $(\oplus_{i\in I} X_i)_p$, with $1 \leq p < \infty$, the ℓ_p-direct sum of the spaces X_i, that is to say :

$$(\bigoplus_{i\in I} X_i)_p = \{\mathbf{x} = (x_i) \in \prod\{X_j, j \in I\} : (\|x_i\|) \in \ell_p\}$$

provided with the norm $\|(x_i)\| = \|(\|x_i\|)\|_p$. If $p = \infty$, then

$$(\bigoplus_{i\in I} X_i)_\infty = \{\mathbf{x} = (x_i) \in l_\infty\left((X_i)\right) : \text{ card (supp } \mathbf{x}) \leq \aleph_0\}$$

*This paper has been partially supported by DGICYT grant PB91-0407.
Received by the editors March 1994
Communicated by J. Schmets
AMS Mathematics Subject Classification : 46A08, 46E40.
Keywords : Unordered Baire-like (UBL) space, Lebesgue-Bochner space, ℓ_p-direct sum.