An Interesting Example for a Three-Point Boundary Value Problem.

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Abstract

Let α, $A \in R$, $\eta \in (0,1)$, and $e(t) \in L^1[0,1]$ be given. Further, let $p(t)$, $q(t)$ be given functions such that $p(t) \geq 0$, $q(t) \geq 0$ for $t \in [0,1]$. This paper concerns the three point boundary-value problem

$$x''(t) = p(t)x(t) + Aq(t)x'(t) + e(t),\ 0 < t < 1, \tag{1}$$

$$x(0) = 0,\ x(1) = \alpha x(\eta). \tag{2}$$

This problem of existence of a solution for this boundary value problem was studied earlier by Gupta, Gupta-Trofimchuk with $p(t) = q(t) = t^{-\frac{1}{4}}$ for various values of α and η. Existence of a solution for this boundary value problem were given for A near zero. When $\alpha = 2$ and $\eta = .6$ Gupta-Trofimchuk were not able to show in [6] that a solution to this boundary value problem exists for any A. In this paper we show that given α, η, there exists an A_1, such that for $A_1 < A < \infty$, the three-point boundary value problem (1)-(2) has a unique solution. Further if $\alpha \leq 1$ then the three-point boundary value problem (1)-(2) has a unique solution for all $A \in R$. This is done as an application of a sharpened existence condition given by the authors earlier for such three-point boundary value problems. The authors made extensive use of computer algebra systems like Maple and MathCad.

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