Periodic boundary value problems for functional differential equations

Tadeusz Jankowski

Abstract

In this paper, the method of quasilinearization has been extended to periodic boundary value problems of nonlinear functional differential equations. It is shown that monotone iterations converge to the unique solution and this convergence is semi–superlinear.

1 Introduction

Put $C_0 = C(J_0, \mathbb{R})$, $C_1 = C(J \times C_0, \mathbb{R})$ with $J_0 = [-\tau, 0]$, $J = [0, T]$ for some $\tau, T > 0$. Let $g \in C_0$ and $g(0) = 0$. We shall study the following periodic boundary value problems for functional differential equations

$$(1) \qquad \begin{cases} x'(t) = f(t, x_t), & t \in J, \\ x(s) = g(s) + x(0), & s \in J_0, \quad x(0) = x(T), \end{cases}$$

where $f \in C_1$, and for any $t \in J$, $x_t \in C_0$ is defined by $x_t(s) = x(t+s)$ for $s \in J_0$. Note that g is given on J_0. If we take $g(s) = 0$ on J_0, then the boundary condition in (1) has the form $x(s) = x(0) = x(T)$, $s \in J_0$.

The differential equation from problem (1) is a very general type. It includes, for example, as special cases, ordinary differential equations if $\tau = 0$, differential–difference equations, and integro–differential equations too.

Received by the editors February 2001.
Communicated by J. Mawhin.
1991 *Mathematics Subject Classification* : 34K10.
Key words and phrases : quasilinearization, periodic boundary value problems, functional differential equations, monotone iterations, semi-superlinear convergence.