

Positive solutions of singular boundary value problems with indefinite weight

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Abstract

Using a generalized version of the method of lower and upper solutions, we prove existence of positive solutions for a class of boundary value problems for a nonlinear equation with singularities whose coefficients change sign.

1 Introduction

In this article we study the existence of positive solutions to a generalized Sturm-Liouville boundary value problem. We consider a second order scalar equation with coefficients that may be of indefinite sign and singular at the end points $t = 0$ and $t = 1$. In this way, we try to combine in a unique setting some features which were previously discussed in various different papers.

Positive solutions for second order nonlinear ordinary and partial differential equations have been widely investigated in the literature. We refer to [3], [11], [17] and the references quoted therein for some classical results in this area, dealing with the Dirichlet problem

$$\begin{aligned} -\Delta u &= \lambda f(x, u), & x \in \Omega, \\ u &= 0, & \text{on } \partial\Omega. \end{aligned} \tag{1}$$

As is well known, the search of radially symmetric solutions to (1) when Ω is an annular domain or a ball in \mathbb{R}^N , yields, up to a suitable rescaling, to the study of a Dirichlet boundary value problem

$$\begin{aligned} -u'' &= \lambda h_1(t, u), \\ u(0) &= 0, \quad u(1) = 0, \end{aligned} \tag{2}$$

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