Recognizing $\mathcal{Q}_{p,0}$ Functions per Dirichlet Space Structure

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Abstract

Under $p \in (0, \infty)$ and Möbius map $\sigma_w(z) = (w-z)/(1-\bar{w}z)$, a holomorphic function on the unit disk $\triangle$ is said to be of $\mathcal{Q}_{p,0}$ class if $\lim_{|w|\to 1} E_p(f, w) = 0$, where

$$E_p(f, w) = \int_\triangle |f'(z)|^2[1 - |\sigma_w(z)|^2]^p dm(z),$$

and where dm means the element of the Lebesgue area measure on $\triangle$. In particular, $\mathcal{Q}_{p,0} = \mathcal{B}_0$, the little Bloch space for all $p \in (1, \infty)$, $\mathcal{Q}_{1,0} = VMOA$ and $\mathcal{Q}_{p,0}$ contains $\mathcal{D}$, the Dirichlet space. Motivated by the linear structure of $\mathcal{D}$, this paper is devoted to: first show that $\mathcal{Q}_{p,0}$ is a Möbius invariant space in the sense of Arazy-Fisher-Peetre; secondly identify $\mathcal{Q}_{p,0}$ with the closure of all polynomials; thirdly characterize the extreme points of the unit closed ball of $\mathcal{Q}_{p,0}$; and finally investigate the semigroups of the composition operators on $\mathcal{Q}_{p,0}$.

Introduction

Let $\triangle$ and $\partial\triangle$ be the unit disk and the unit circle in the finite complex plane $\mathbb{C}$. Denote by $\mathcal{H}$ the set of functions holomorphic on $\triangle$, endowed with the topology of the compact-open (i.e. the uniform convergence on compact subsets of $\triangle$). The

*Project supported by the AvH-Foundation of Germany as well as the NNSF (19771006) and SEC of China.

Received by the editors February 2000.

Communicated by J. Delanghe.

1991 *Mathematics Subject Classification* : 30D45, 32A37, 46E15, 47B33.

Key words and phrases : $\mathcal{Q}_{p,0}$, Möbius invariance, polynomial density, extreme points, composition semigroups.