Large minimal covers of $\mathrm{PG}(3, q)$

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Abstract

A *cover* of $\Sigma = \mathrm{PG}(3, q)$ is a set of lines S such that each point of Σ is incident with at least one line of S. A cover is *minimal* if no proper subset is also a cover. We study minimal covers of Σ which are 'large'; the main results being constructions of sets of this kind and an upper bound on the size of minimal covers.

1 Introduction

A *cover* of $\Sigma = \mathrm{PG}(3, q)$ is a set S of lines such that every point of Σ is on at least one line of S. A cover is said to be *minimal* if no proper subset is also a cover. Every cover of Σ contains at least $q^2 + 1$ lines, and the covers with exactly this many lines are the spreads. In [1], A. Blokhuis *et. al.* study covers of Σ (and of finite generalized quadrangles) which are 'small'. In essence, they give a structure theorem for minimal covers S with $q^2 + 1 < |S| < q^2 + q + 1$.

In this note we study 'large' minimal covers. A natural first problem is to find the maximal size of a minimal cover. We begin by finding an upper bound on the size of these sets, proceed to give some constructions for large minimal covers, and finally discuss some connections between this problem and others outstanding in the literature. Along the way we describe an interesting 'regular' cover.

Throughout, $\mathrm{star}(P)$ denotes the set of lines of Σ on a point $P \in \Sigma$ while $\mathrm{pen}(P, \pi)$ denotes the plane pencil of lines defined by the incident point-plane pair (P, π).

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